The numbers (x.x.x) refer to the number problems in Ash and Ash (1986). They refer to chapter, section, and problem. These are selected problems that cover the breadth of topics in our review. Solutions are given in the back of Ash and Ash (1986) and will be posted so that you can selfgrade this homework.

(1.1.1) Let $f(x) = 2 - x^2$ and $g(x) = (x-3)^2$. Find

(a) $f(0)$  
(b) $f(1)$  
(c) $f(b^3)$

(d) $g(0)$  
(e) $g(1)$  
(f) $g(b^3)$

(g) $(g(b))^3$  
(h) $f(2a+b)$  
(i) the range of $f$ and $g$, if the domain is $(-\infty, \infty)$

(1.1.4) Let $f(x) = 2x+1$. Does $f(x)$ ever equal $(f(a))^2$?

(1.2.4) Suppose $f$ is an increasing function. If $x$ decreases, what does $f(x)$ do?

(1.2.8) A function $f$ is said to be even if $f(-x) = f(x)$ for all $x$; for example, $f(7) = 3$ and $f(-7) = 3$, $f(-4) = -2$ and $f(4) = -2$, and so on. A function $f$ is said to be odd if $f(-x) = -f(x)$ for all $x$; for example, $f(3) = -12$ and $f(-3) = +12$, $f(-6) = -2$ and $f(6) = +2$, and so on. The functions $\cos x$ and $x^2$ are even, $\sin x$ and $x^3$ are odd, and $2x+3$ and $x^3 + x$ are neither.

The figure shows the graph of a function $f(x)$ for $x \geq 0$.

(a) If $f$ is even, complete (sketch) the graph for $x \leq 0$.

(b) Complete the graph if $f$ is odd.

(1.2.9) Find $f(x)$ if the graph of $f$ is the line $AB$ where $A = (1,2)$ and $B = (2,5)$.

(1.3.1) Convert from radians to degrees.

(a) $\pi/5$  
(b) $5\pi/6$  
(c) $-\pi/3$

(1.3.2) Convert from degrees to radians.

(a) $12^\circ$  
(b) $-90^\circ$  
(c) $100^\circ$

(1.3.5) Let $\sin x = a$, $\cos y = b$, and evaluate the expression in terms of $a$ and $b$, if possible.

(a) $\sin(-x)$  
(b) $\cos(-y)$  
(c) $-\sin x$

(d) $-\cos y$  
(e) $\sin^2 x$  
(f) $\sin x^2$

(1.4.2) Find the inverse of the function by inspection, if it exists.

(a) $x - 3$  
(b) Int $x$

(c) $1/x$  
(d) $-x$

Note Int $x$ is the greatest integer function, the largest integer that is less than or equal to $x$.

(1.5.1) Arrange each set of numbers from smallest to largest without using a calculator or math software.

(a) $e^{10}$, $e^{10}$, $e^{10}$

(b) $e^{1/2}$, $e^{1/3}$, $e^{3}$, $e^{5}$, $e^{6}$

(c) $e^{10}$, $e^{10}$, $e^{10}$

(1.5.10) Solve
(a) \(2 e^{-x} - 3 = 0\)  
(b) \(\ln (2x + 7) = -1\)  
(c) \(e^x = 5\)  
(d) \(e^{2x+7} > -5\)  
(e) \(-\ln x = 4\)  
(f) \(\arcsin e^x = \pi/6\)  
(g) \(\ln (-x) = 4\)  
(h) \(\ln x + 2 \ln x = 3\)  
(i) \(\ln (5x + 3) = \ln 2x\)  
(j) \(\ln (5x + 3) = \ln 2x\)  
(k) \(4 \ln x + \ln 2x = 3\)  
(l) \(\ln (5x - 3) = \ln 2x\)  
(m) \(e^x = e^{-x}\)  
(n) \(x \ln x = 0\)  
(o) \(x e^x + 2 e^x = 0\)  
(p) \(\frac{25}{2 + \ln 3x} = 5\)  
(q) \(\frac{x + 5}{x + 4}\)  
(r) \(\frac{1}{x^2 - 4} > 0\)  
(s) \(\frac{x + 2}{x^2 - 4}\)  
(t) \(\ln x + 2 \ln x = 3\)  
(u) \(\ln (-x) = 4\)  
(v) \(\ln x + 2 \ln x = 3\)  
(w) \(e^{2x+7} > -5\)  
(x) \(-\ln x = 4\)  
(y) \(\arcsin e^x = \pi/6\)  
(z) \(\ln (-x) = 4\)  
(A) \(\lim x \to 3 x^2\)  
(B) \(\lim x \to \infty x^2\)  
(C) \(\lim x \to 0 \cos x\)  
(D) \(\lim x \to -\infty \tan^{-1} x\)  
(E) \(\lim x \to \infty (1/3)^x\)  
(F) \(\lim x \to \infty \sqrt{x}\)  
(G) \(\lim x \to \frac{\pi}{2} \tan x\)  
(H) \(\lim x \to 0 \sin x\)  
(I) \(\lim x \to 2 (x^2 + 3x - 1)\)  
(J) \(\lim x \to \infty (x^2 + 3x - 1)\)  
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(W) \(\lim x \to \infty \ln x\)  
(X) \(\lim x \to \infty (x^2 + 3x - 1)\)  
(Y) \(\lim x \to \infty \ln x\)  
(Z) \(\lim x \to \infty (x^2 + 3x - 1)\)  
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(Y) \(\lim x \to \infty \ln x\)  
(Z) \(\lim x \to \infty (x^2 + 3x - 1)\)
(a) \( \lim_{x \to 3} f(x) = 5 \) \hspace{1cm} (b) \( \lim_{x \to 3^-} f(x) = 6 \) and \( \lim_{x \to 3^+} f(x) = \infty \)

Note: If a function \( g(x) \) has a discontinuity at \( x = a \) in a function \( g(x) \), the discontinuity is called \textit{removable} if we can define or redefined \( g(a) \) to make the function continuous. Jump and infinite discontinuities are not removable.

(3.2.1) If the curve in the figure is the graph of function \( f(x) \), estimate \( f'(0), f'(-100), f'(100) \). Sketch the graph of \( f'(x) \).

(3.2.3) Let \( y \) be the distance (in feet) from a submerged water bucket up to the top of a dug well at time \( t \) (in seconds). Suppose \( dy/dt = -2 \) at a particular instant. Which way is the bucket moving, and how fast is it moving?

(3.2.4) If \( dy/dx \) is positive, how does \( y \) change if \( x \) decreases?

(After 3.2.6) \(^2\) Let \( f(x) \) represent the number of shrubs located along an experimental land-surface transect, in the interval \([0,x]\) where \( x \) is measured in meters. For example, if \( f(100) = 20 \) then 20 shrubs lie along the interval \([0,100]\).

(a) What does \( f(x+\Delta x) - f(x) \) represent in this context?

(b) What does the quotient \( \frac{f(x + \Delta x) - f(x)}{\Delta x} \) represent?

(c) What does \( f'(x) \) represent?

(d) What values of \( f'(x) \) are impossible?

Note \([a,b]\) is a \textit{closed interval}, \((a,b)\) is an \textit{open interval}, while \([a,b]\) is closed on the left and open on the right.

(3.2.9) Which of the following is necessarily true, necessarily false, or could be true but is ambiguous?

(a) If \( f(2) = g(2) \), then \( f'(2) = g'(2) \).

(b) If \( f \) is increasing, then \( f' \) is increasing.

(c) If \( f \) is a \textit{periodic} function, that is, if \( f \) repeats every \( b \) units, then \( f' \) is also periodic.

(d) If \( f \) is even, then \( f' \) is even.

(3.2.13) Let \( f(t) \) be the temperature in your city at time \( t \). If it is uncomfortably hot at time \( t=2 \), are you pleased or displeased with the indicated data?

(a) \( f'(2) = 6 \) and \( f''(2) = -4 \) \hspace{1cm} (b) \( f'(2) = 6 \) and \( f''(2) = -6 \) \hspace{1cm} (c) \( f'(2) = 0 \)

(3.3.1) Find

(a) \( D_x x^6 \) \hspace{1cm} (f) \( \frac{d(x^{2/3})}{dx} \)

(b) \( D_x (1/x^6) \) \hspace{1cm} (g) \( D_t 0 \)

(c) \( D_x x^{9/7} \) \hspace{1cm} (h) \( \frac{d(e^t)}{dt} \)

\(^2\) The word “after” refers to a revised version of a problem in Ash and Ash (1986), or to a “new” problem modeled after a similar problem in that reference. In this case, the solution in the back of Ash and Ash may only partially apply or not apply at all.
(d) $D_x \sqrt[3]{u}$

(i) $D_x 4$

(e) $\frac{d}{dx} \left( \frac{1}{\sqrt{x}} \right)$

(3.3.5) If $u = \tan t$, find $du/dt$.

(3.3.7) If $f(x) = 1/x^4$ find $f'(17)$.

(3.3.9) Differentiate the function.

(a) $x^{-3}$
(b) $x^{14}$
(c) $\sqrt{x^5}$
(d) $1/x^5$
(e) $x$
(f) $\ln x$

(g) $x^{-1/3}$
(h) $x^4$
(i) $1/x^4$
(j) $1/x$
(k) $1/x^2$

(3.3.13) If $a=b^4$, find $da/db$ and $db/da$ directly and verify that $\frac{da}{db} = \frac{1}{\frac{db}{da}}$

(3.3.15) Find the slope at (-2,16) on the graph of $y = x^4$ and find the equations of lines tangent and perpendicular to the graph at that point.

(3.5.3) Differentiate (a, b, etc not included)

(e) $\frac{x^3 + x}{3}$
(f) $2x^3 \cos x$
(g) $\sqrt{x} \ln x$

(h) $\sec x \tan x$
(j) $2e^x + \ln x$
(k) $4x^2 \tan^{-1} x$

(3.5.6) Find

(a) $\frac{d(xe^x)}{dx}$
(b) $\frac{d^2(xe^x)}{dx^2}$
(c) $\frac{d^3(xe^x)}{dx^3}$
(d) $\frac{d^n(xe^x)}{dx^n}$

(3.5.12) Use the second derivative to find the concavity of (a) $y = \sin x$ and (b) $y = x^3$ (rest of problem omitted).

(3.5.17) If $y - x \sin x$, show that $y'' + y = 2 \cos x$.

(3.6) Find the derivative of the (composite) function.

1. $e^{6x}$
2. $\sin 2x$
3. $e^x$
4. $-e^x$
5. $\sin^{-1}(3-x)$
6. $2 \cos 5x$
7. $5x e^{2x}$
8. $\frac{1}{2 + x}$
9. $x^3(2x+5)^6$
10. $\ln (5-x)$

11. $\tan^{-1} x/2$
12. $\exp(1/x)$
13. $\sin x^4$
14. $\sqrt{x^2 + 4x}$
15. $\ln \sin e^x$
(3.7.1a) Use implicit or logarithmic differentiation of the functions to find $dy/dx$ for $x + y = y \tan y + x \tan x$

(3.7.2) Find $dy/dx$ and $dx/dy$ for $y = \cos (x^2 + y^2)$

(3.7.8) Differentiate the function
(a) $2^x$  
(b) $x^4$  
(c) $x \sin x$  
(e) $(2x + 3)^4$

(3.8.1) Find
(a) $\int 3 \sin x \, dx$  
(b) $\int \sin 3x \, dx$  
(c) $\int u^4 \, du$  
(f) $\int x^{-1} \, dx$

(3.8.2) Find $f(x)$ if $f'(x) = \sin x + x^2$ and $f(0) = 10$.

(3.8.5) Find $y$ if $y' = 2x + 3$ and $y = -2$ when $x = 1$.

(3.8) Find the antiderivative for the function, if possible, using basic methods.

<table>
<thead>
<tr>
<th>9. $\frac{3}{3-x}$</th>
<th>11. $(x^2 + 5)^{1/2}$</th>
<th>12. $5/x$</th>
<th>13. $1/5x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16. $7 \cos \pi x$</td>
<td>17. $\cos x^3$</td>
<td>18. $\frac{x^2 + 6}{5}$</td>
<td>21. $\sqrt{2 + \frac{1}{4} x}$</td>
</tr>
<tr>
<td>24. $\frac{\sin x}{6}$</td>
<td>27. $3e^x$</td>
<td>31. $\pi$</td>
<td>34. $\frac{x^3}{2}$</td>
</tr>
</tbody>
</table>

Con’t, perform the indicated antidifferentiation, if possible, using basic methods.

<table>
<thead>
<tr>
<th>37. $\int \frac{1}{\sqrt{t}} , dt$</th>
<th>40. $\int \frac{1}{x} , dx$</th>
<th>45. $\int dx$</th>
<th>49. $\int \ln x , dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>51. $\int \frac{1}{1-v} , dv$</td>
<td>52. $\int \frac{2}{3+4x} , dx$</td>
<td>53. $\int \frac{1}{\sin x} , dx$</td>
<td>56. $\int \frac{5t+3}{2} , dt$</td>
</tr>
</tbody>
</table>

(3.R) Con’t with …

| 59. $\int \frac{1}{3-t} \, dt$ | 60. $\int \frac{1}{\sqrt{3-t}} \, dt$ | 61. $\int \sqrt{1+2x} \, dx$ | 62. $\int \sqrt{1+2x^2} \, dx$ |

(4.1.1) Use (i) the first derivative test and (ii) the second derivative test to locate the relative maxima and minima.

(a) $f(x) = x^3 - 3x^2 - 24x$  
(b) $x^4 - x^2$  
(c) $x \ln x$

(4.1.2) Locate the relative maxima and minima, if possible, with the given information.

(a) $f'(2) = 0$, $f''(x) < 0$ for $1.9 < x < 2$, $f'(x) > 0$ for $2 < x < 2.001$

(e) $f'(2) = 0$, $f''(2) = 6$  
(f) $f'(2) = 0$, $f''(2) = 0$
(4.2.1) Find the maximum and minimum values of \( f(x) \) on the indicated interval.

\[
\begin{align*}
(a) \quad & f(x) = x^3 + x^2 - 5x - 5 \\
& \text{(i) } (-\infty, \infty) \quad \text{ (ii) } [0,2] \quad \text{ (iii) } [-1,0] \\
(c) \quad & f(x) = \frac{x-2}{x^2 - 3} \\
& \text{(i) } [0,5] \quad \text{ (ii) } [2,5]
\end{align*}
\]

Reminder \([a,b]\) is a closed interval, \((a,b)\) is an open interval, while \([a,b)\) is closed on the left and open on the right.

(4.2.2) Suppose \( f'(x) \) is always negative. Find the largest and smallest values of \( f \) on \([3,4]\).

(4.2.8) Let \( f(x) = -x^3 - 5x^2 - 13x + 4 \); find the maximum and minimum slope on the graph of \( f \) for \( 0 \leq x \leq 1 \).

(4.3.2) Find (consider using L’Hopital’s rule and orders of magnitude)

\[
\begin{align*}
(a) \quad & \lim_{x \to \infty} \frac{x^2}{\ln x} \\
(e) \quad & \lim_{x \to 0} \frac{\sin x - x}{\cos x - 1} \\
(f) \quad & \lim_{x \to \infty} \frac{e^{-x}}{1 + e^{-x}}
\end{align*}
\]

(4.4.1) Find \( \lim_{x \to \infty} xe^{1/x} \) as (a) \( x \to \infty \), (b) \( x \to 0^- \), (c) \( x \to -\infty \).

(4.4.4) Sketch (do not use a graphing calculator or computer software) the function \( xe^{1/x} \) near \( x = 0 \) after finding the limits as \( x \to 0^+ \) and \( x \to 0^- \).

(4.5) Sketch the graph of the function \( f(x) \) without using a graphing calculator or computer software. Use the following guidelines: First, determine the ends of the graph (e.g., using limits). Second, determine if there are any gaps and their location. Third, find relative extrema, by finding critical numbers and classifying them as relative maxima, relative minima, or neither. Fourth, determine concavity of the function and use it to decide where the function is concave up (\( f'' \) positive) or down (\( f'' \) negative). Use hints from familiar graphs. It gives you a head start.

\[
\begin{align*}
1. \quad & -x^2 + 4x + 5 \\
7. \quad & \sin (2x - \pi/6) \\
15. \quad & \frac{x-1}{x+1}
\end{align*}
\]

(4.6) (Related rates)

4. Water flows at 8 cubic feet per minute into a cylinder with radius 4. How fast is the water level rising?

10. A stone is dropped into a lake, causing circular ripples whose radii increase at 2m/s. How fast is the disturbed area growing when the outer ripple has radius 5.

VERY IMPORTANT ASSIGNMENT:

(4.7) Use Newton’s method and continue until two successive approximations agree to the indicated number of decimal places. Then check the accuracy by searching for a sign change in \( f(x) \).

1. Find \( \sqrt{38} \) by solving \( x^2 = 39 \) for the positive value of \( x \). Use \( x = 6 \) as the initial guess and stop after agreement to two decimal places.

2. Solve \( e^x = 3 - x^2 \); 3 decimal places. Begin by sketching the graphs of \( e^x \) and \( 3 - x^2 \) on the same set of axes. Examine their intersections to determine the number and approximate value of solutions.

(4.8.1) Find the differential
(4.8.2) Find \(dy\) if \(y = 2x^3 + 3\)

(4.9.1) Separable differential equations. Solve

- (a) \(y' = -x \sec y\)
- (d) \(y' = \frac{y}{2x^3 + 3}\)
- (e) \(x^2 \, dy = e^y \, dx\)

(4.9.2) Find the particular solution satisfying the given condition

- (a) \(y' = xy, \, y(1) = 3\)
- (b) \(yy' + 5x = 3, \, y(2) = 4\)

(4.9.2) (a) Solve \(xy' = 2\) and sketch (by hand) the family of solutions. (b) Find the particular solution of the family through the point (2,3)

(5.2.2) Use integrals to express the area between the graph of \(y = \ln x\) and the \(x\)-axis for

- (a) \(1 \leq x \leq 5\)
- (b) \(1/2 \leq x \leq 1\)

(5.2.3a) Decide which is the larger of the pair of integrals

\[\int_{0}^{3} x^2 \, dx, \quad \int_{1}^{3} x^2 \, dx\]

(5.2.4) Decide if the integral is positive, negative, or zero.

- (a) \(\int_{0}^{3\pi/2} \cos x \, dx\)
- (b) \(\int_{0}^{2\pi} \cos^2 x \, dx\)

(5.2.5a) True or false? If \(f(x) < 0\) for all \(x \in [a,b]\) then \(\int_{a}^{b} f(x) \, dx < 0\).

(5.2.7a) Let \(A_1 = \int_{a}^{b} f(x) \, dx\). Consider the area and translation to decide which of the following is equal to \(A_1\):

\[A_2 = \int_{a+3}^{b+3} f(x) \, dx, \quad A_3 = \int_{a+3}^{b+3} f(x+3) \, dx, \quad A_4 = \int_{a+3}^{b+3} f(x-3) \, dx\]

(5.3) Evaluate the integral.

1. \(\int_{-1}^{1} (6x^2 - 3x + 2) \, dx\)
2. \(\int_{\pi/3}^{\pi/2} \sin 2x \, dx\)
3. \(\int_{0}^{1} \frac{1}{1+x^2} \, dx\)
4. \(\int_{0}^{1/2} \sin nx \, dx\)
5. \(\int_{2}^{5} dx\)
6. \(\int_{0}^{3} e^{x^3} \, dx\)

(5.3.23) Find the average value of \(\sin x\) on \([0, \pi]\)

(5.3.24) Find (a) \(\int_{0}^{1} x^3 \, dx\) and (b) \(\int_{1}^{2} x^3 \, dx\)

(5.4.1a) Approximate (by-hand) the integral using Simpson’s rule with the given number of subintervals.
A \sin x \cos x \int_{0}^{41} dx , n = 4

(soln. not in Ash and Ash) Check your approximate answer by comparing to the exact solution.

(5.6) Improper integrals. Evaluate the integrals

1. \int_{3}^{\infty} \frac{1}{x^5} dx

4. \int_{-1}^{0} \frac{1}{x^2} dx

7. \int_{-\infty}^{0} \frac{1}{1+x^2} dx

12. \int_{0}^{\infty} \sin x \, dx

13. \int_{-\infty}^{\infty} e^{-x^4} \, dx

15. \int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} \, dx

Re 15, given \( F(x) = \frac{1}{2} \left( \frac{x}{x^2+1} + \arctan x \right) \)

(After 6.1.1) A particle of contamination is being carried by a flowing fluid. If the velocity of the particle along a flowline is a constant \( v \) (m/d), then after time \( t \) (d) it will have traveled a distance \( vt \) (m). Suppose instead that the velocity is not constant, but is proportional to \( t^2 \) at time \( t \). For example, the velocity at time 3 days is 9 m/d, the velocity at time 3.1 d is 9.61 m/d, and so on. Using integration, find the total distance traveled between times 3 and 5d.

(6.1.8) If the specific heat of an object of unit mass is constant, then the heat needed to raise its temperature is given by

\[ \text{Heat} = (\text{specific heat}) \times (\text{desired increase in temperature}) \]

For example, if the object has specific heat 2 and its temperature is to be raised from 72º to 78º F, then 12 calories of heat are needed. Suppose that the specific heat of the object is not constant, but is given by the cube of the object’s temperature. Thus, the object becomes harder and harder to heat as its temperature increases. Using integration, find the heat needed to raise its temperature from 54º to 61º F.

(6.1.25) Snow starts falling at time \( t=0 \), and then falls at the rate of \( R(t) \) flakes/hour at time \( t \). (a) How much snow will accumulate by time 10? (b) Some of the flakes melt after they land, and don’t live to see time 10. Suppose that only \( \frac{1}{4} \) of the newly landed flakes still exist 3 hours later, only \( \frac{1}{5} \) still exist 4 hours later and, in general, of \( F \) newly fallen flakes, only \( F/(x+1) \) flakes will last \( x \) more hours. How much snow accumulates by time \( t=10 \)?

(After 6.1.26) If water discharges at a rate \( Q \) (m³/d) through a column of porous media of cross-sectional area \( A \) (m²) and length \( L \) (m), then the total flow resistance \( R \) that it encounters is proportional to \( L/A \). (Aside, \( R= L/AK \), where \( K \) is hydraulic conductivity; assume unit \( K \), i.e., \( K=1 \text{m/d} \).) Suppose the water is injected at the center of a sphere of radius 10m, through a hole of radius 1m at its center, and water flows spherically (radially) out of the hole and through the porous sphere. (Aside, this is the simplest model of a cavity injection well.) The formula \( L/A \) doesn’t apply directly because the current encounters spherical “cross sections” (see figure), with increasing area with radial distance, rather than constant area \( A \); e.g., visualize the “flow” away from the center of an onion, through the concentric onion shells. Use spherical “shells” to find an integral formula for \( R \) in spherical porous flow.

(6.3.1.a) Find the area of the region with the indicated boundaries. \( y = x^2, y = 3x \)

(6.3.2a) Find the area of the shaded region in the sketch.

(6.3.4) Use an integral to find the distance between the two points \( C = (x_1, y_1) \) and \( D = (x_2, y_2) \).

(6.5.1) Find an explicit formula for integral \( I(x) \) if

\[ I(x) = \int_{2}^{x} (t + 5) \, dt \]
(6.5.3) Integral with a variable upper limit: Suppose it begins raining at 3pm and $x$ hours later it is raining at the rate of $x^3$ inches per hour. For example, at 3:30 pm it is raining at the rate of $1/8$ inch per hour. (a) Find the total rainfall by 5 pm. (b) Find the cumulative rainfall after $x$ hours.

(6.5.6) Let $f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ \frac{1}{x} & \text{if } x > 1 \end{cases}$ and let $I(x) = \int_0^x f(t) \, dt$

(a) Find $I(1/2)$  
(b) Find $I(2)$  
(c) Find $I(x)$, in general, for $x \geq 0$.

(6.5.7) Let $I(x) = \int_0^x \ln(t) \, dt$ and $J(x) = \int_0^{x/2} \ln(t) \, dt$. (a) Which is the larger of $I(7)$ and $J(7)$? (b) How do the graphs of $I(x)$ and $J(x)$ compare with each other?

(6.5.8) Find (do not look up) the derivative of integral functions, (a) $\frac{d}{dx} \text{Erf} x$  
(b) $\frac{d}{dx} \text{Ei} x$  
(c) $\frac{d^2}{dx^2} \text{Ei} x$

Note: these derivatives are commonly encountered in hydrology. For example, (a) gives the dispersive flux for step change in concentration or temperature, (b) gives the groundwater flux toward a pumping well, and (c) gives the rate of change of that flux.

(6.R.7c) Let $I(x) = \int_x^1 f(t) \, dt$.

Find an explicit formula for $I(x)$ if $f(x)$ has the graph:

(7.2) Use substitution for antidifferentiation of

1. $\int xe^{x^2} \, dx$  
2. $\int x \sqrt{3x^2 + 7} \, dx$  
8. $\int \frac{1}{x \ln x} \, dx$  
18. $\int \sin^2 \pi x \, dx$  
21. $\int \ln(2x + 3) \, dx$

(7.2.23) We know that $\int \frac{1}{1+x^2} \, dx = \arctan x$. Is the following antidifferentiation correct?

$$\int \frac{1}{1+3x^2} \, dx = \int \frac{1}{1+(3\sqrt{x})^2} \, dx = \arctan \sqrt{3}x + C?$$

(7.4.2) Use partial fractions to decompose

(a) $\frac{12}{x^2 - 3}$  
(b) $\frac{5x}{(x^2 + 1)(x - 2)}$

(7.4.4) Using partial fractions find

(a) $\int \frac{2x + 3}{x^2 - 4x + 4} \, dx$  
(b) $\int \frac{8x}{x^3 - 1} \, dx$

(7.5.1) Use integration by parts to derive the formula given in tables for
(a) \[ \int xe^x \, dx \]  
(b) \[ \int \tan^{-1} x \, dx \]

(7.5.2) Use integration by parts to find

(a) \[ \int x^2 e^x \, dx \]  
(b) \[ \int x \tan^{-1} x \, dx \]

(7.7.1) Use trigonometric substitution to find

(a) \[ \int \frac{dx}{\sqrt{x^2 - a^2}} \]  
(c) \[ \int \frac{\sqrt{a^2 + x^2}}{x} \, dx \]

(7.8) Outline a method for finding each antiderivative.

(a) \[ \int \sin \left( \frac{x}{\sqrt{x}} \right) \, dx \]  
(b) \[ \int \frac{1}{\sqrt{4 - x^2}} \, dx \]  
(c) \[ \int \frac{1 + 2x}{1 + x^2} \, dx \]

(7.9.1) For each integral, perform the indicated substitution, and then stop after reaching an integral involving only \( u \).

(b) \[ \int \sin(\ln x) \, dx \] , \( u = \ln x \)  
(c) \[ \int \frac{\sqrt{x^2 - 4}}{x^2} \, dx \] , \( u \) is the angle indicated in the figure

(7.9.3c) Show that the integrals are equal without evaluating them.

\[ \int_{2a}^{2b} \sqrt{\frac{x}{2}} \, dx \] and \[ 2 \int_{a}^{b} \sqrt{\sin x} \, dx \]

(8.1.1) Write out the first three terms of the series.

(a) \[ \sum_{n=3}^{\infty} (-1)^n \frac{1}{2n+1} \]  
(b) \[ \sum_{n=1}^{\infty} n^2 a_n \]

The following require increasingly sophisticated convergence tests. Consult your calculus book for examples and guidelines.

(8.2) Decide if the geometric series converges or diverges. If the series converges, find it sum.

(2) \[ \frac{1}{4} - \frac{1}{16} + \frac{1}{64} - \frac{1}{256} + ... \]  
(5) \[ \sum_{n=3}^{\infty} 4^{-n} \]

(8.3) Decide if the geometric series converges or diverges.
(b) \(1 - \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \ldots\) 
(j) \(\sum \frac{1}{2^n n!}\)

(8.4) Decide if the geometric series converges or diverges.

(8) \(-\frac{1}{4} - \frac{2}{9} - \frac{3}{16} - \frac{4}{25} + \ldots\) 
(4) \(\sum \frac{1}{3^{n-1}}\)

(8.5.4) Decide if the alternating series converges or diverges.

(a) \(\frac{3}{2} - \frac{4}{3} + \frac{5}{4} - \frac{6}{5} + \ldots\) 
(b) \(\sum_{n=3}^{\infty} (-1)^{n+1} \frac{n^2}{n!}\)

(8.6) For each power series what is the interval of convergence?

(1) \(\sum_{n=3}^{\infty} (-1)^n (n+1)x^n\) 
(5) \(x - x^2 + x^3 - x^4 + \ldots\)

(8.7.1) Find a power series for each function, and find the interval of convergence of the series.

(a) \(\sqrt{1+x}\) 
(d) \(\frac{1}{2 - 3x}\)

(8.8.) Find a power series representation of each function using, in particular, a Maclaurin series approach. We didn’t have time to review this in class. But it is straightforward. Review in your calculus book.

(a) \((1 + x)^6\) 
(b) \(\ln(1 + x)\)

The following two problems deal with power series in powers of \(x-b\), i.e., a Taylor Series, which is of the form.

\[\sum_{n=0}^{\infty} a^n (x-b)^n = a_0 + a_1(x-b) + a_2(x-b)^2 + a_3(x-b)^3 + \ldots\]

Consult your calculus book.

(8.10.1) Find the interval of convergence for the power series (in \(x-4\), it’s a Taylor Series) of \(\sum \frac{(x-4)^n}{n 3^n}\)

(8.10.3) Find the (Taylor) series expansion and its interval of convergence.

(a) \(\ln x\) in powers of \(x-1\) 
(b) \(\frac{1}{x}\) in powers of \(x\)