Dynamic Earthquake Triggering Due to Stress from Surface Wave Particle Displacement

Introduction

Earthquakes can be triggered by other earthquakes. This can be due to local stress field changes by nearby earthquakes, known as static triggering, or by stresses from the passage of seismic waves from a large, remote earthquake [Velasco et al., 2008]. Static triggering is the type that generates aftershocks, as stress is redistributed once the main shock ruptures [Jagla, 2010]. van der Elst and Brodsky [2010] name three possible stresses that have been proposed as triggering signal transmitters: “coseismic static strain changes, progressive postseismic strain changes (including afterslip and viscous creep), and dynamic strains from radiated seismic waves”. They go further to identify possible mechanisms as “direct Coulomb frictional failure, reduction in fault strength, and pore fluid pressure changes” [van der Elst and Brodsky, 2010].

As my research involves far-field triggering of earthquakes, I am interested in further understanding the process of remote triggering. Remote triggering will not occur from changes due to coseismic static strain, as these stresses decrease with distance from the fault. These are changes in strain that take place on a fault when an earthquake releases stress on another fault. However these changes are useful in explaining near-field triggering rather simply for some time, as the strain changes permanent [van der Elst and Brodsky, 2010]. Afterslip and creep also fall off with distance, and so will also not have a role in triggering of distant earthquakes. Although after many years viscous deformation can travel large distances, it does not explain far-field triggering that occurs in much shorter time periods, which is what I want to explore [van der Elst and Brodsky, 2010].

Since the above two stresses fall off further from the main shock, dynamic stress changes are the only explanation for distant triggering. This is evidenced by the tendency of the triggered earthquakes to occur at the arrival of surface waves [van der Elst and Brodsky, 2010; Velasco et al., 2008]. The arrivals of these waves, and therefore the displacements due to the waves, create stress changes in the far-field that promote damages along faults that reduce the friction along the fault [Gonzalez-Huizar and Velasco, 2011; Jagla, 2011]. For this paper I will look at Gonzalez-Huizar and Velasco’s [2011] development of a stress tensor that would be accumulated by Rayleigh and Love wave particle displacements. Since they do not explicitly reveal their strain values and stress tensors, I will be deriving them based on the displacement equations used. To model stress changes due to these displacements, they apply a synthetic 20s period Rayleigh wave and a synthetic 20s Love wave (both in the fundamental mode) and determine equations for triggering potential based on fault type. While generally the peak dynamic stress is measured by calculating a multiple of the maximum value on a velocity seismogram, this does not take into account the depth or fault plane orientation [Gonzalez-Huizar and Velasco, 2011]. Gonzalez-Huizar and Velasco [2011] define these two neglected parameters as important factors in determining triggering stress conditions. They believe from other studies that
“triggered earthquakes are caused by the unclamping of faults that are in a preferred orientation relative to the passing seismic waves” [Gonzalez-Huizar and Velasco, 2011]. However, this process is not well understood and in need of more study, as evidenced by the many explanations behind dynamic triggering [Velasco et al., 2008; Gonzalez-Huizar and Velasco, 2011].

**Methods**

To determine the dynamic stress tensor for the passage of surface waves, Gonzalez-Huizar and Velasco [2011] start with the particle displacements for each wave (Rayleigh and Love) in a Possion half-space.

**Rayleigh Wave Stresses**

Rayleigh waves are a combination of P and SV waves that propagate in the x-z plane [Stein and Wysession, 2003]. Gonzalez-Huizar and Velasco [2011] begin with the particle displacement equations for the x and z components in a halfspace, which they call $U_1$ and $U_3$ respectively, adapted from Stein and Wysession [2003]:

$$U_1 = A k_1 \sin(\omega t - k_1 x_1) [\exp(B k_1 x_3) - C(D k_1 x_3)], \quad (1)$$

$$U_3 = A k_1 \cos(\omega t - k_1 x_1) [(B k_1 x_3) + E \exp(D k_1 x_3)], \quad (2)$$

where $A$ is amplitude, $\omega$ is angular frequency, $k_1$ is the horizontal wave number, and $B$, $C$, $D$, and $E$ are constants: $B = -0.85$, $C = 0.58$, $D = -0.39$, and $E = 1.47$. Gonzalez-Huizar and Velasco [2011] refer to the x, y, z coordinates as $x_1$, $x_2$, $x_3$. From these particle displacements, we can get strain values from

$$\varepsilon_{ij} = \varepsilon_{ji} = \frac{1}{2} (U_{uj} + U_{uj}) \quad (3)$$

[Aster Notes, 2011; Stein and Wysession, 2003; Gonzalez-Huizar and Velasco, 2011]. Since Gonzalez-Huizar and Velasco [2011] do not explicit give what the different strain components are, I have worked them out for the purposes of this paper. Since “Rayleigh waves exhibit shearing and compressional/dilatational particle motion” and do not have motion in the $U_2/y$ direction [Gonzalez-Huizar and Velasco, 2011], I have worked out the 11, 33, and 13 = 31 components:

$$\varepsilon_{11} = -A k_1^2 \cos(\omega t - k_1 x_1) [\exp(B k_1 x_3) - C(D k_1 x_3)], \quad (4)$$

$$\varepsilon_{33} = A k_1 \cos(\omega t - k_1 x_1) [(B k_1) + E D k_1 \exp(D k_1 x_3)], \quad (5)$$

$$\varepsilon_{13} = \varepsilon_{31} = \frac{1}{2} (A k_1 \sin(\omega t - k_1 x_1) [B k_1 \exp(B k_1 x_3) - C D k_1]$$

$$- A k_1^2 \sin(\omega t - k_1 x_1) [(B k_1 x_3) + E \exp(D k_1 x_3)]), \quad (6)$$

where $\varepsilon_{11}$ is the derivative of $U_1$ with respect to $x_1$, $\varepsilon_{33}$ is the derivative of $U_3$ with respect to $x_3$, and $\varepsilon_{13}/\varepsilon_{31}$ is half of the sum of the derivative of $U_1$ with respect to $x_3$ and $U_3$ with respect to $x_1$. From the strain calculations, we can get the stress values of the same components from
\[ \sigma_{ij} = \lambda \Theta_{ij} + 2\mu \varepsilon_{ij}, \]  

(7)

where

\[ \Theta = \varepsilon_{ii} = U_{ii} = U_{i} + \frac{\partial U_{1}}{\partial x_{1}} + \frac{\partial U_{2}}{\partial x_{2}} + \frac{\partial U_{3}}{\partial x_{3}}, \]  

(8)

and \( \lambda \) and \( \mu \) are Lamé parameters \([\text{Aster Notes, 2011; Stein and Wysession, 2003}]. \) Since there is no motion in the \( x_2 \) direction, \( \varepsilon_{22} \), or \( \frac{\partial U_{2}}{\partial x_{2}} \), will fall out of equation (8). For this situation, we have:

\[ \sigma_{11} = \lambda(\varepsilon_{11} + \varepsilon_{33}) + 2\mu \varepsilon_{11}, \]  

(9)

\[ \sigma_{33} = \lambda(\varepsilon_{11} + 3\varepsilon_{33}) + 2\mu \varepsilon_{33}, \]  

(10)

\[ \sigma_{13} = \sigma_{31} = 2\lambda \varepsilon_{13}. \]  

(11)

\[ \text{Gonzalez-Huizar and Velasco [2011] assume a Poisson solid, which means } \lambda = \mu \; [\text{Aster Notes, 2011; Stein and Wysession, 2003}], \text{ so we can simplify equation (9), (10), and (11) to only use one Lamé parameter:} \]

\[ \sigma_{11} = \lambda(3\varepsilon_{11} + \varepsilon_{33}) \]  

(12)

\[ \sigma_{33} = \lambda(\varepsilon_{11} + 3\varepsilon_{33}) \]  

(13)

\[ \sigma_{13} = \sigma_{31} = 2\lambda \varepsilon_{13}. \]  

(14)

\[ \text{[Gonzalez-Huizar and Velasco, 2011]. Again, I have worked out the 11, 33, and 13/31 components using equations (4), (5), and (6) (above):} \]

\[ \sigma_{11} = \lambda(-3A k_1^2 \cos(\omega t - k_1 x_1)) \left[ \exp(B k_1 x_3) - C(D k_1 x_3) \right] \]  

\[ + A k_1 \cos(\omega t - k_1 x_1) \left[ (B k_1 + E D k_1 \exp(D k_1 x_3)) \right], \]  

(15)

\[ \sigma_{33} = \lambda(-A k_1^2 \cos(\omega t - k_1 x_1)) \left[ \exp(B k_1 x_3) - C(D k_1 x_3) \right] \]  

\[ + 3A k_1 \cos(\omega t - k_1 x_1) \left[ (B k_1 + E D k_1 \exp(D k_1 x_3)) \right], \]  

(16)

\[ \sigma_{13} = \sigma_{31} = \lambda(A k_1 \sin(\omega t - k_1 x_1)) \left[ B k_1 \exp(B k_1 x_3) - C D k_1 \right] \]  

\[ - A k_1^2 \sin(\omega t - k_1 x_1) \left[ (B k_1 x_3) + E \exp(D k_1 x_3) \right]. \]  

(17)

**Love Wave Stresses**

Love waves are a result of interactions of SH waves and cannot occur in a half-space because they require a velocity structure that varies with depth \([\text{Stein and Wysession, 2003}]. \) Therefore, we consider displacement in a layer over a half-space, using the \( y \)-direction, or transverse, displacement:

\[ U_2 = A \exp(i(\omega t - k_2 x_1)) \cos(k_3 y), \]  

(18)

where
\[ r_\beta = (c^2/\beta^2 - 1)^{1/2} \]  

Once again, from equation (3) we can obtain strain values. Since Love waves only induce shearing and displacement is in the \(x_2\) direction, I have worked out strain values for the \(12 = 21\) and \(23 = 32\) components:

\[ \varepsilon_{12} = \varepsilon_{21} = -\frac{1}{2} i A k_1 \exp \left( i(\omega t - k_1 x_1) \right) \cos(k_1 r_\beta x_3), \]

\[ \varepsilon_{23} = \varepsilon_{32} = -\frac{1}{2} A k_1 r_\beta \exp \left( i(\omega t - k_1 x_1) \right) \sin(k_1 r_\beta x_3), \]

where the strain only depends on the derivatives of \(U_2\) with respect to \(x_1\) and \(x_3\) as there is no Love wave displacement in the \(x_1\) or \(x_3\) directions. From equation (7) we find that

\[ \sigma_{12} = \sigma_{21} = 2\mu \varepsilon_{12}, \]

\[ \sigma_{23} = \sigma_{32} = 2\mu \varepsilon_{23} \]

Using equations (20) and (21) I find the stress for these two components to be

\[ \sigma_{12} = \sigma_{21} = -iA k_1 \mu \exp(\imath(\omega t - k_1 x_1)) \cos(k_1 r_\beta x_3) \]

\[ \sigma_{23} = \sigma_{32} = -A k_1 r_\beta \mu \exp(\imath(\omega t - k_1 x_1)) \sin(k_1 r_\beta x_3) \]

To look at the stresses generated by passing Rayleigh and Love waves in a general manner, Gonzalez-Huizar and Velasco [2011] plotted the stress for a 20s period wave as a function of depth from 0 to 30km and time from 0 to 25s.

**Model stress effects on fault orientations**

Thus far we have calculated stress tensors as a function of depth and time, \(T^0(d,t)\). However, an important factor in dynamic triggering is in the orientation of the wave propagation relative to the fault orientation. To do this, Gonzalez-Huizar and Velasco [2011] rotate their stress tensor relative to the fault orientation (Figure 1) and around the angles \(\alpha\) and \(\theta\) by multiplying the stress tensor by the Euler angles. Euler angles are three angles that can be used to describe the orientation of a rigid body and therefore are three successive angles of rotation [Goldstein, 1950]. As shown in Figure 2a, we start with the \(xyz\) axes and rotate them by an angle of \(\phi\) around the \(z\) axis, counterclockwise. The new coordinate system is \(\xi\eta\zeta\). In Figure 2b, we rotate this new system counterclockwise around \(\xi\) by an angle of \(\theta\). We call this system of coordinates \(\xi\eta\zeta'\). In Figure 2c, we have rotated the system a third and final time around \(\zeta'\), counterclockwise by an angle of \(\psi\), giving the final coordinate system \(x'y'z'\) [Goldstein, 1950]. From these successive rotations, we can describe the rotation of a matrix, \(x\) to \(x'\) can be described as

\[ x' = Ax \]

where \(A\) is the product of three matrices:

\[ A = BCD \]
[Goldstein, 1950; Weisstein, 2011]. From Goldstein [1950] and Weisstein [2011], the matrices $B$, $C$, and $D$ relate to each angle of rotation:

$$D = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(28)

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}$$

(29)

$$B = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ 

(30)

Using the above matrices to define $A$ in equation (27),

$$A = \begin{bmatrix} \cos \psi \cos \phi - \cos \theta \sin \phi \sin \psi & -\sin \psi \cos \phi - \cos \theta \sin \phi \cos \psi & \sin \theta \sin \phi \\ \cos \psi \sin \phi + \cos \theta \cos \phi \sin \psi & -\sin \psi \sin \phi + \cos \theta \cos \phi \cos \psi & -\sin \theta \cos \phi \\ \sin \theta \sin \psi & \sin \theta \cos \psi & \cos \theta \end{bmatrix}$$

(31)

[Goldstein, 1950; Weisstein, 2011]. From Figure 1, we can see that Gonzalez-Huizar and Velasco [2011] rotate their system around $z$ by $\alpha$, and then around $\xi'$ by $\theta$. Therefore the Euler angles for rotating their stress tensors are $\phi = \alpha$, $\theta = \theta$, and $\psi = 0$. Since Gonzalez-Huizar and Velasco [2011] do not use a third rotation, I take the third angle to be zero. Since again, the authors do not explicitly say what the stress tensors are based on the displacement equations and do not explicitly show what the rotation matrix, $A$, is for the situation, I find that for the stress tensor rotations, $A$ becomes

$$A = \begin{bmatrix} \cos \alpha & -\cos \theta \sin \alpha & \sin \theta \sin \alpha \\ \sin \alpha & \cos \theta \cos \alpha & -\sin \theta \cos \alpha \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

(32)

Because $\cos \psi$ will be one, and $\sin \psi$ will be zero. By multiplying the stress tensors $\hat{T}^0(d,t)$ by $A$, we get $T(d,t,\alpha,\theta)$ with stresses corresponding to the fault plane orientation [Gonzalez-Huizar and Velasco, 2011]. Taking the stress tensors as $x$ in equation (26), I calculate the new stress tensors to be

$$T_b(d,t,\alpha,\theta) = AT_b^0(d,t) = \begin{bmatrix} \cos \alpha & -\cos \theta \sin \alpha & \sin \theta \sin \alpha \\ \sin \alpha & \cos \theta \cos \alpha & -\sin \theta \cos \alpha \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} \sigma_{11} & 0 & \sigma_{13} \\ 0 & 0 & 0 \\ \sigma_{31} & 0 & \sigma_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{11}\cos \alpha + \sigma_{31}\sin \theta \sin \alpha & \sigma_{11}\sin \alpha - \sigma_{31}\sin \theta \cos \alpha & \sigma_{31}\cos \alpha \\ 0 & \sigma_{13}\sin \theta \sin \alpha - \sigma_{33}\sin \theta \cos \alpha & \sigma_{33}\cos \alpha \\ \sigma_{31}\cos \alpha + \sigma_{33}\sin \theta \sin \alpha & \sigma_{13}\sin \alpha - \sigma_{33}\sin \theta \cos \alpha & \sigma_{33}\cos \theta \end{bmatrix},$$

(33)

$$T_l(d,t,\alpha,\theta) = AT_l^0(d,t) = \begin{bmatrix} \cos \alpha & -\cos \theta \sin \alpha & \sin \theta \sin \alpha \\ \sin \alpha & \cos \theta \cos \alpha & -\sin \theta \cos \alpha \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} 0 & \sigma_{12} & 0 \\ \sigma_{21} & 0 & \sigma_{23} \\ 0 & \sigma_{32} & 0 \end{bmatrix}$$

(34)
where $\mathbf{T}_R$ is the stress tensor for the Rayleigh waves and $\mathbf{T}_L$ is the stress tensor for the Love waves. Refer to equations (15), (16), (17), (24), and (25) for $\sigma_{ij}$ values. As shown in Figure 1, there are now components of stress oriented with the new system of coordinates, $\delta \sigma_n$, $\delta \tau_d$, and $\delta \tau_s$ which are the components acting on the normal, dip, and strike directions of the fault, respectively [Gonzalez-Huizar and Velasco, 2011].

To look at how stress from passing Love waves varies with these angles ($\alpha, \theta$), Gonzalez-Huizar and Velasco [2011] plot $\delta \sigma_n$, $\delta \tau_d$, and $\delta \tau_s$ (from $T_{33}$, $T_{11}$, and $T_{22}$ of the rotated stress tensor, respectively) for a 20s period Love wave at 5 km depth and 5s as a function of varying $\alpha$ and $\theta$ angles.

### Triggering Potential

The triggering potential is given as the change of the Coulomb failure function

$$\delta \text{CFF} = \delta \tau + \delta \tau + \mu \delta \sigma_n,$$

where $\delta \tau$ and $\delta \sigma_n$ are the changes on the fault’s shear and normal stresses, and $\mu$ is the coefficient of friction [Gonzalez-Huizar and Velasco, 2011]. If $\delta \text{CFF}$ is positive the fault becomes closer to failure and if negative the fault moves away from failure. The potential is dependent on the faulting mechanism (i.e. what type of fault the stress is acting on). For example, $\delta \tau_d$ will have opposing signs for normal and reverse faults. Similarly for strike-slip faults, $\delta \tau_s$ will have opposing signs depending on the relative motion. Since $\delta \sigma_n$ represents the change in normal stress on the fault, it will indicate unclamping if it is positive. Assuming pure dip-slip motion along the normal and reverse faults, Gonzalez-Huizar and Velasco [2011] define the triggering potentials as

$$P(\text{reverse}) = \delta \tau_d + \mu \delta \sigma_n$$

$$P(\text{normal}) = -\delta \tau_d + \mu \delta \sigma_n$$

$$P(\text{strike-slip, left-lateral}) = \delta \tau_s + \mu \delta \sigma_n$$

$$P(\text{strike-slip, right-lateral}) = -\delta \tau_s + \mu \delta \sigma_n.$$

To add oblique movement to the faults, I would add a $\pm \delta \tau_s$ to equations (36) and (37).

Gonzalez-Huizar and Velasco [2011] plot the triggering potentials for each fault type for a 20s period Love wave at 5 km depth and 5s, with varying $\alpha$ and $\theta$. They also plotted triggering potentials as a function of $\alpha, \theta$, and time for both 20s period Love and Rayleigh waves at 5 km depth. The times chosen corresponded to minimum and maximum displacements and zero displacements for the $U_2$ and $U_3$ directions, respectively.

### Discussion

Before rotating the stress tensors, Gonzalez-Huizar and Velasco [2011] plotted 20s period Rayleigh and Love waves as a function of depth and time (Figure 3) in order to determine which stress tensor components play an important role at a given depth and time. Recall that positive normal stresses are dilatational and negative are compressional. In Figure 3 we can see that the stresses change with time, fluctuating between minimum and maximum values and also changes direction. For example,
at 10s the normal stresses for the Rayleigh wave (Figure 2a) are at a maximum while the shear stress is at 0MPa. At 5s we see the opposite effect. Depth also plays an important role. $\sigma_{11}$ for the Rayleigh wave and $\sigma_{12}$ for the Love wave both have maximums at the surface. Gonzalez-Huizar and Velasco [2011] explain this as at shallow depths, “Rayleigh wave’s normal stresses and Love wave’s horizontal shear stress mainly act on planes perpendicular to the direction of wave propagation”. They go on to say “at greater depths, normal stress and lateral shearing is stronger on horizontal planes” as is evidenced by $\sigma_{13}$ and $\sigma_{23}$ increasing with depth [Gonzalez-Huizar and Velasco, 2011]. Note also that the maximum stresses due to Rayleigh waves are more than three times that of the Love waves.

Figure 4 shows stress changes of a 20s period Love wave at 5s and 5km depth as a function of fault orientation ($\alpha=0$ to $180^\circ$ and $\theta=0$ to $90^\circ$). Stress changes reach maximums in the direction normal to the fault plane and in the strike direction. Shear stress in the dip direction does not see large changes. Although Love waves only induce shearing, Gonzalez-Huizar and Velasco [2011] draw attention to the fact that Love waves do cause a change in the normal stress on a rotated plane. Note that if the fault was not rotated ($\alpha=0^\circ$), the normal stress would remain constant ($\delta\sigma_n = 0$MPa). They find a maximum normal stress for a fault plane striking $45^\circ$ (meaning $\alpha=45^\circ$) from propagation direction and dipping vertically ($\theta=90^\circ$) [Gonzalez-Huizar and Velasco, 2011]. Based on the rotated stress tensors developed, if we know the orientation of a fault, we can model the stress changes that will take place on the fault for any Rayleigh or Love wave and determine the potential for triggering on that fault.

Figure 5 shows triggering potentials for a 20s period Love wave at 5s and 5km depth for each fault type. Gonzalez-Huizar and Velasco [2011] find that a right-lateral strike slip fault is the most likely to be triggered by this 20s period Love wave, specifically one striking at $70^\circ$ from wave propagation and dipping vertically. However, we also see similar maximum triggering potentials for vertical left-lateral strike-slip faults striking at $\sim 35^\circ$ (estimated from Figure 5) and normal faults striking at $\sim 80$ to $90^\circ$ from the wave propagation and dipping at $\sim 45^\circ$. A reverse fault does not reach the maximum triggering potentials that the other faults reach, but has its own maximum value at a vertical fault striking at $\sim 50^\circ$ from wave propagation. The above orientations are the most likely to trigger for each fault type.

Figure 6 is the triggering potential as a function of time for a 20s period Love wave at $5$ km depth. At the minimum and maximum displacements, $U_2$, the potential for triggering is at zero for all fault types. The maximum triggering potentials for all fault types are actually achieved at inflection points on the displacement plot, when the displacements in the $U_2$ direction are relatively close to zero. Therefore, when Love waves reach a time when their particle displacements are at an inflection point, all fault types can be triggered, if the fault is at the conducive orientation. Figure 7 also shows triggering potentials, but for a 20s period Rayleigh wave through time at 5km depth, looking at time relative to vertical motion ($U_3$). In this case, the highest triggering potentials are achieved at maximum vertical displacements for all fault types. Intermediate triggering potentials are found at inflection points along the time-displacement curve, and negative triggering potentials are found at minimums in vertical displacement. Therefore, for faults at certain orientations, triggering can occur when Rayleigh waves reach a time when their vertical particle displacements reach a maximum. It is important to note that in both cases, the maximum triggering potentials do not necessarily mean the fault will fail, but that the fault is being moved towards failure. This failure could occur immediately, or if the triggering potential (change in stress) is not large enough, can occur later.

Applying their model to the Australian Bowen Region, a region of low seismicity that has been found to have dynamically triggered earthquakes in previous studies, Gonzalez-Huizar and Velasco [2011] find that Love waves arriving $45^\circ$ from the local compressional stress are most likely to trigger earthquakes. Looking at 8 large events that triggered seismicity, 7 events had incident angles relative to the local compressional stress orientation within $10$ to $12^\circ$ from $\alpha = 45^\circ$ or $-45^\circ$. However, they admit themselves that they assumed all the faults were in the same orientation in the absence on detailed orientation information and the model was based on a wave with very specific characteristics [Gonzalez-
To better understand dynamic triggering in an area, they admit that they need more information on the local stress state and more information about the seismic waves themselves.

**Conclusions**

Models of stress generated by surface waves and triggering potentials based on those stress changes are potentially useful, in that we can determine the potential for a fault to fail and a triggered earthquake to occur knowing fault orientation and surface wave characteristics. For shallow depths, we find that $\sigma_{11}$ for Rayleigh waves and $\sigma_{12}$ for Love waves reach a maximum, while at larger depths lateral shearing for horizontal planes is dominant ($\sigma_{13}$, $\sigma_{15}$, and $\sigma_{23}$ increase with depth). 20s period Love waves see a maximum change in normal stress for vertical faults striking at 45°. Modeling triggering potentials as functions of fault orientation, depth and time can give us an indication if the surface waves are moving the fault towards or away from failure, and potentially tell us if a fault at a given orientation is likely to be triggered by a particular wave. We also find that for Love waves, maximum triggering potentials are reached when the displacement in the $U_2$ direction is at an inflection point on a time-displacement curve. On the contrary, for Rayleigh waves this occurs when vertical displacement is at a maximum. While Gonzalez-Huizar and Velasco [2011] find that their model predictions do correlate with real dynamic triggering data, they admit that more information about the local stress state must be included in the model. Also, specific fault orientations and specific characteristics of the seismic waves need to be known.

**Figures**

*Figure 1. From Figure 1, Gonzalez-Huizar and Velasco [2011]. With an incoming seismic wave $T^0$, (a) is the orientation of the fault plane, in the coordinate system $T$, and (b) is the stress changes due to the passing seismic wave on the fault plane in the normal, strike and dip directions ($\delta\sigma_n$, $\delta\tau_s$ and $\delta\tau_d$).*
Figure 2. From Figure 4-6, Goldstien [1950]. Euler Angles: (a) is a rotation counterclockwise around $z$ by $\phi$, giving the new axes $\xi\eta\zeta$. (b) is a counterclockwise rotation around $\xi$ by $\theta$, giving the new axes $\xi'\eta'\zeta'$. (c) is the final counterclockwise rotation around $\zeta''$ by $\psi$, giving the final coordinate system $x'y'z'$. 

Figure 3. From Figure 2, Gonzalez-Huizar and Velasco [2011]. (a) corresponds with a Rayleigh wave. The top is a synthetic vertical displacement for a 20s period Rayleigh wave. Below it is the related particle displacement. Below these are the normal stresses and shear stress as a function of time and depth. (b) corresponds with a Love wave. The top is a synthetic transverse displacement for a 20s period Love wave. Below it is the related particle displacement. Below these are the shear stresses as a function of time and depth.
**Figure 4.** From Figure 3, Gonzalez-Huizar and Velasco [2011]. Modeling dynamic stress of a 20s period Love wave on a fault of arbitrary orientation \((\alpha, \theta)\) for the stress components in the strike, dip, and normal directions \((\delta \tau_S, \delta \tau_D, \text{ and } \delta \sigma_n)\).

**Figure 5.** From Figure 4, Gonzalez-Huizar and Velasco [2011]. Triggering potentials, \(P\), of a 20s period Love wave on reverse, normal, left-lateral strike-slip, and right-lateral strike-slip faulting as a function of fault orientation \((\alpha, \theta)\).
Figure 6. From Figure 5, Gonzalez-Huizar and Velasco [2011]. Top is displacement, $U_2$, through time for a 20s period Love wave. Below are triggering potentials, $P$, as a function of fault orientation ($\alpha, \theta$) for key moments of transverse motion (inflection points, minimum, and maximum) for different fault types (reverse, normal, left-lateral, right-lateral).
Figure 7. From Figure 6, Gonzalez-Huizar and Velasco [2011]. Top is displacement, $U_3$, through time for a 20s period Rayleigh wave. Below that is triggering potential, $P$, as a function of fault orientation ($\alpha, \theta$) for key moments for vertical displacement (inflection points, minimum, maximum) for different fault types (reverse, normal, left-lateral, right-lateral).
References

Aster, R. (2011), Class Notes: *Stress, Strain, and Elasticity* (1) and *Surface Waves in Layered Media* (4).


