**Basis Functions**

**Linear Systems**

\[ y(t) = \phi[x(t)] \]

Output:
\[ \phi[x(t) + x(t)] = \phi[x(t)] + \phi[x(t)] \]

Scaling:
\[ \phi[a \cdot x(t)] = a \cdot \phi[x(t)] \]

**Convolution Relationship:**
\[ \phi[0] = 0 \]

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**Time Invariance**

If \( \phi[x(t)] = y(t) \)

Then \( \phi[x(t - \tau)] = y(t - \tau) \)

Consistency

If \( x(t) = 0 \) for \( t < 0 \)

Then \( \phi[0] = 0 \) \( t = 0 \)

\( \Rightarrow \) if a system is linear, then the relationship between \( x(t) \) and \( y(t) \) is characterized by an integral equation \( \Rightarrow \) Convolution Integral.

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**Delta Function**

\[ \delta(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{otherwise} \end{cases} \]

**Properties of the Delta Function:**

\[ \int_{-\infty}^{\infty} f(t) \delta(t - a) \, dt = f(a) \]

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**Input Response of a (linear) System**

\[ \Rightarrow \text{output } \phi[x(t)] \text{ when } x(t) = \delta(t) \]

\[ y(t) = \phi[\delta(t)] \]

\( \Rightarrow \) Next: We show that output of any linear time-invariant system can be written as a convolution.

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