Homework 3 - Spring 2008: Earth 441 – Aquifer Mechanics

Assigned: Feb 7, 2008 Due: Feb 14, 2008 at the beginning of class

Please make sure to show all your work, including any diagrams you used, in order to get full credit. This may be submitted in either paper or electronic format (Shasta@nmt.edu). Please cite your sources.

1. (25 pts) Darcy Columns - The two Darcy columns pictured below have the diameters (d) as described. The pressure head at the top is 20 cm water and the pressure head at the bottom is 250 cm water. Assume K is the same for both columns and equal to $2 \times 10^{-4}$ m/s. Assume porosity is equal to 0.15. Assume the diameter of column 1 is uniform. The diameter of each section of column 2 is shown on the diagram.

Use the following numbers for the lengths of each section on column 1:
A – 120 cm, B – 50 cm, C – 80 cm

Use the following numbers for the lengths of each section on column 2:
1 – 30 cm, 2 – 70 cm, 3 – 20 cm, 4 – 50 cm, 5 – 80 cm

Required:

a) Find the flux rate (Q), the specific discharge (q), and seepage velocity (v) for column 1.
b) Find the flux rate (Q), the specific discharge (q), and seepage velocity (v) for column 2.
2. **Storage in confined aquifers (12 pts)**

**Background:** The specific storage in a confined aquifer is given by:

\[ S_s = \rho_w g (\alpha + n\beta) \]  

(1)

where \( \rho_w \) is the density of water, \( g \) is the gravitational constant, \( \alpha \) is the vertical compressibility of the rock matrix, and \( \beta \) is the compressibility of water. The property \( \alpha \) may vary in space (heterogeneous aquifer material) but is assumed to not vary in time.

During pumping a confined aquifer has four possible responses:

i. water expands
ii. the aquifer skeleton compresses elastically
iii. the individual grains of the aquifer skeleton compress
iv. the aquifer consolidates (i.e., the aquifer grains are rearranged inelastically)

**Required:**

(a) Which of these responses are represented by the term \( \alpha \)? What assumption(s) is made about the aquifer and water system in order to allow \( \alpha \) to be considered a constant with respect to time?

(b) Which of these aquifer responses are represented by the term \( \beta \)? What assumption(s) is made about the aquifer and water system in order to allow \( \beta \) to be considered a constant?

(c) Which of these aquifer responses are not represented in Equation 1? What assumption(s) is made to disregard these aquifer responses?

(d) In the derivation of Equation 1 (see the class lecture notes), it is asserted that in most hydrologic applications we can assume that \( dV_w = -dV_T \), where \( dV_w \) is the change in water volume in a control volume, and \( dV_T \) is the change in total volume of a control volume. What is the basis of this assumption?

3. **Storage in a confined aquifer (12 pts)**

**Background:**

The storativity of a confined aquifer is \( 8 \times 10^{-4} \). It has a porosity of 0.25 and a thickness of 300 m. Assume the temperature is 25ºC.

**Required:**

(a) How many cubic meters of water are removed from storage under an area of \( 1 \times 10^6 \) m\(^2 \) when the head declines 1 m? Assume that the aquifer remains confined and that the storativity remains the same during water removal.

(b) What is the vertical compressibility (find the value of \( \alpha \) in Equation 1) for this aquifer?

(c) If the aquifer matrix was incompressible, what would the storativity be?
4. **Storage in unconfined and confined aquifers** (16 pts)

**Background:**

In a ground-water basin of 12 square miles, there are two aquifers: an upper unconfined aquifer 500 ft in initial saturated thickness and a lower confined aquifer with an available hydraulic head drop of 150 ft (before it becomes unconfined). Hydraulic tests have determined that the specific yield of the upper unit is 0.12 and the storativity of the lower unit is $4 \times 10^{-4}$. What is the amount of recoverable ground water in the basin?

Source: Schwartz and Zhang, 2003, Problem 4.3

5. **Specific retention, specific yield, and porosity** (10 pts)

**Background:**

A cube of wet sediment measures 10.0×10.0×10.0 cm and has a mass of 2.00 kg. After the sediment sample is drained by gravity, the mass of the sample is 1.78 kg. After the sample is oven-dried, the mass is 1.75 kg.

**Required:**

(a) Calculate the specific retention.
(b) Calculate the specific yield.
(c) Calculate the porosity of the sample as a percentage.

6. **Darcy Column – Varying Hydraulic Conductivity** (10 pts)

**Background:**

A vertical Darcy column open to the atmosphere at both ends, contains three different porous media in layers of equal thickness perpendicular to flow. The column is 60 cm long, and 20 cm in diameter. Layer 1 has a hydraulic conductivity of $5 \times 10^{-5}$ m/s, layer 2 has a hydraulic conductivity of $1 \times 10^{-6}$ cm/s. The flux of water through the column is $9.4 \times 10^{-10}$ m$^3$/s.
Required:
What is the hydraulic conductivity of the third layer? Given the three K values what do you think the three porous media most likely are (assuming that they are unconsolidated sediments (not rocks))?
**Non-Credit Questions (Practice):** Recommended for anyone who got this type of problem wrong on the first homework. This is not required, but if you answer, I will look at your answer and “grade” it so you can practice.

1. For the following Darcy column, plot the **total head, elevation head, and pressure head** as a function of distance.

![Diagram of Darcy column](image1)

2. For the column in the next figure, graph the elevation head, the pressure head, and the total head as a function of distance.

Figure 1: Column. There is a free water surface at each end of the column.