A Reading List in Inverse Problems

Brian Borchers

Draft of January 13, 1998

This document is a bibliography of books, survey articles, and on-line documents on various topics related to inverse problems. I've tried to avoid listing research papers, because there are far more research papers on each of these topics than I could ever hope to include in this bibliography. Hopefully, the material that I have included in this bibliography will point the reader to important research papers in each topic.

I've annotated the bibliography with an indication of the mathematical level of each item. The coding is "Mx", where x=1 means that the item doesn't require any mathematics beyond calculus and linear algebra, x=2 means upper level undergraduate mathematics (differential equations, probability theory, numerical analysis, Fourier transforms, etc.), and x=3 means graduate level (most often this means functional analysis.)

References

[Bau87]   Johann Baumeister. *Stable Solution of Inverse Problems.* Vieweg, Braunschweig, 1987.

> M3. Theory of ill-posed problems, singular value decomposition. Tikhonov regularization, least squares solution of systems of linear equations, convolution equations, final value problems, parameter identification. No exercises.

[Ber91]   James G. Berryman. Lecture notes on nonlinear inversion and tomography: I. borehole siesmic tomography. http://sepwww.stanford.edu/sep/berryman/NOTES/lecture_notes.html, October 1991.

> M2. Seismic inversion, traveltime inversion, tomography. Exercises.

[BG70]   G. Backus and F. Gilbert. Uniqueness in the inversion of inaccurate gross earth data. *Philosophical Transactions of the Royal Society A*, 266:123–192, 1970.

        M2.5. This is a classic paper introducing the method of
Backus and Gilbert for linear inverse problems.

[Car87]   Philip Carrion. *Inverse Problems and Tomography in Acoustics and Seismology.* Penn Publishing Company, Atlanta, Georgia, 1987.

        M2.5. Seismic inversion and tomography. No exercises.

[Cla85]   Jon F. Claebourt. *Imaging the Earth's Interior.* Blackwell Scientific Publications, Palo Alto, CA, 1985. Out of print, but available on the web. See http://sepwww.stanford.edu/sep/prof/index.html.

        M2. Seismic Inversion. No exercises. Claerbout has several other books available from the same web site.

[CPC87] Steven C. Constable, Robert L. Parker, and Catherine G. Constable. Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data. *Geophysics*, 52(3):289–300, 1987.

        M2. This paper describes an iterative method for solving a nonlinear discrete inverse problem by linearizing the problem and using the discrepancy principle.

[EHN96] Heinz W. Engl, Martin Hanke, and Andreas Neubauer. *Regularization of Inverse Problems.* Kluwer Academic Publishers, Dordrecht, 1996.

        M3. Examples, ill-posed linear operator equations, Tikhonov regularization, iterative regularization methods, the conjugate gradient method, numerical implementation, nonlinear problems. No exercises.

[Eng93]   Heinz W. Engl. Regularization methods for the stable solution of inverse problems. *Surveys on Mathematics for Industry*, 3:71–143, 1993.

        M2.5. A broad survey of regularization methods with lots of examples.

[Gro93]   Charles W. Groetsch. *Inverse Problems in the Mathematical Sciences.* Vieweg, Bruanschweig; Wiesbaden, 1993.

        M2. An introduction to the mathematics of inverse problems, requiring only undergraduate mathematics. Integral equations of the first kind, parameter estimation in differential equations, regularization, iterative methods, the maximum entropy method, the Backus-Gilbert method. Exercises.

[Han92] Per Christian Hansen. Analysis of discrete ill-posed problems by means of the L-curve. *SIAM Review*, 34(4):561–580, December 1992.

M2. This paper discusses the use of the L-curve criteria for selecting a regularization parameter and compares it with generalized cross validation.

[Han94] Per Christian Hansen. Regularization tools: A MATLAB package for analysis and solution of discrete ill–posed problems. *Numerical Algorithms*, 6:1–35, 1994. http://www.imm.dtu.dk/documents/users/pch/Regutools/regutools.html.

M2. The regularization toolbox provides a variety of functions for solving inverse problems, including the SVD and generalized SVD, truncated SVD solutions, Tikhonov regularization, maximum entropy regularization, and a variety of examples.

[Han98] Per Christian Hansen. *Rank–Deficient and Discrete Ill-Posed Problems*. SIAM, Philadelphia, 1998.

M2. Numerical methods for discretized inverse problems, including basic theory, direct and iterative methods for regularization, and methods for picking the regularization parameter. Examples drawn from the author's regularization toolbox. No Exercises.

[HH93] Martin Hanke and Per Christian Hansen. Regularization methods for large–scale problems. *Surveys on Mathematics for Industry*, 3:253–315, 1993.

M2.5. Another survey paper, with special emphasis on iterative methods for large scale problems. Contains a good discussion of methods for selecting the regularization parameter.

[Kir96] Andreas Kirsch. *An Introduction to the Mathematical Theory of Inverse Problems*. Springer Verlag, New York, 1996.

M3. Regularization theory, Tikhonov regularization, regularization by discretization, the method of Backus and Gilbert, inverse eigenvalue problems, inverse scattering problems. Includes exercises.

[LH74] C. L. Lawson and R. J. Hanson. *Solving Least Squares Problems*. Prentice-Hall, Englewood Cliffs, New Jersey, 1974.

M2. A classic book on the solution of linear least squares problems. Contains an early discussion of the L-curve. This book has recently been republished by SIAM. Exercises.

[Lin88] Laurence R. Lines, editor. *Inversion of Geophysical Data.* Society of Exploration Geophysicists, 1988.

    M2. A collection of reprints of tutorial/survey papers on inversion. Linearized inversion techniques, seismic inversion, inversion of electromagnetic and potential field data. No exercises

[Men89] William Menke. *Geophysical Data Analysis: Revised Edition*, volume 45 of *International Geophysics Series.* Academic Press, San Diego, 1989.

    M2. Generalized inverses, maximum likelihood methods, the method of Backus and Gilbert, nonlinear inverse problems, numerical algorithms, applications. No exercises.

[Mor84] V. A. Morozov. *Methods for Solving Incorrectly Posed Problems.* Springer-Verlag, New York, 1984.

    M3. Pseudoinverses, regularization, methods for picking the regularization parameter, nonlinear problems. No exercises.

[Par94] Robert L. Parker. *Geophysical Inverse Theory.* Princeton University Press, Princeton, NJ, 1994.

    M2. Mathematical background, examples from geophsyics, Tikhonov regularization, resolution, nonlinear problems. Exercises.

[Roy91] Dilip N. Ghosh Roy. *Methods of Inverse Problems in Physics.* CRC Press, Boca Raton, 1991.

    M2.5. Inverse problems in physics, the Povzner-Levitan transform, the Gelfand-Levitan Equation, Jost Functions, the Marcenko integral equation, the Radon transform. No exercises.

[Sab87] P.C. Sabatier, editor. *Inverse Problems: An Interdisciplinary Study.* Academic Press, London, 1987.

    M2.5. A collection of papers, mostly applications in tomography, electromagnetic inverse scattering, quantum mechanics, and other areas. No exercises.

[Sca97] John Scales. Theory of seismic imaging. http://landau.Mines.EDU/ samizdat/imaging/index.html, 1997.

    M2. Seismic inversion, Kirchoff migration, ray tracing, finite difference methods. Exercises.

[SS97]   John Scales and Martin Smith. DRAFT: Geophysical inverse theory. http://landau.Mines.EDU/ samizdat/inverse_theory/, 1997.

> M2. Presents a Bayesian approach to inverse theory. Exercises.

[Str88]   Gilbert Strang. *Linear Algebra and its Applications*. Harcourt Brace Jovanovich Inc., Fort Worth, third edition, 1988.

> M1. This is a textbook on linear algebra at a slightly higher level than the typical introductory course. Contains a good discussion of the SVD. Exercises.

[TA77]   A. N. Tikhonov and V. Y. Arsenin. *Solutions of Ill-Posed Problems*. John Wiley, New York, 1977.

> M3. The definitive treatment of the theory of Tikhonov regularization.

[Tar87]   Albert Tarantola. *Inverse Problem Theory: Methods for Data Fitting and Model Parameter Estimation*. Elsevier, New York, 1987.

> M3. This book describes a Bayesian approach to discrete and continuous inverse problems.

[Two77]   S. Twomey. *Introduction to the Mathematics of Inversion in Remote Sensing and Indirect Measurements*. Elsevier, Amsterdam, 1977.

> M1.5. Mathematical background, quadrature, Tikhonov regularization, examples. No exercises. This book has recently been republished by Dover.

[Var79]   J. M. Varah. A practical examination of some numerical methods for linear discrete ill-posed problems. *SIAM Review*, 21:100–111, 1979.

> M2. This paper compares a variety of methods for solving discrete linear inverse problems.

[Wah90]   Grace Wahba. *Spline Models for Observational Data*. SIAM, Philadelphia, 1990.

> M3. Although this book is largely about fitting splines to data, it also contains a very useful discuss of cross validation. No exercises.

[Win91]   G. Milton Wing. *A Primer on Integeral Equations of the First Kind*. SIAM, Philadelphia, 1991.

> M2. This book provides an introduction to theory and methods for the practical solution of integral equations of the first kind.