Spring, 2011 Theoretical Seismology (GEOP 523)  
Math Overview  
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This set of notes covers mathematical tools that we will need to be familiar with in this course, and complements the appendix in Shearer’s book.

**Complex Numbers.** We will make extensive use of complex numbers in this course. You need to be thoroughly familiar with plotting and performing various operations (length, angle, division, multiplication, etc.) on complex numbers.

**Linear Algebra.** Many of the quantities that we will deal with are multidimensional. We will generally denote such quantities as bold, upper-case letters for matrices, e.g., $U$, and bold, lower-case letters for vectors, e.g., $x$. Components are identified using subscripts, e.g., $U_{ij}$, or $u_x$. You are expected to be thoroughly familiar with basic the properties and operations of vector-matrix equations, e.g. $y = Ax$, such as dot ($\cdot$) and cross ($\times$) products, and eigenvalue-eigenvector concepts.

You should understand the diagonalization of the symmetric matrix $A$ as

$$A = U \Lambda U^T$$

where the orthonormal matrix of unit-length eigenvectors is

$$U = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}$$

and the diagonal vector of associated eigenvalues is

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}.$$

We will utilize the *Einstein summation convention* where repeated indices indicate summation, e.g.,

$$A_{ij}x_j \equiv \sum_j A_{ij}x_j = Ax.$$

Note that one can always discern the number of dimensions in a quantity in index notation, e.g., $A_{ij}x_j$ is a vector because of the implied summation over index $j$. 

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You should also be familiar with index objects such as the Kronecker delta

\[ \delta_{ij} = \begin{cases} 1 & \text{if } i = j; \\ 0 & \text{if } i \neq j. \end{cases} \]

We will go to lengths to develop a physical understanding of important second-order tensors in this class, such as stress and strain.

**Vector Calculus.** Vector calculus operators that you should be familiar with include:

- **Gradient:** \( \nabla f(x) = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z} \)
  
is a vector field that measures the direction of steepest descent from, and is normal to, constant contours of some scalar field, \( f \) at \( x \).

- **Divergence:** \( \nabla \cdot g(x) = \frac{\partial g_x}{\partial x} + \frac{\partial g_y}{\partial y} + \frac{\partial g_z}{\partial z} = \partial_i g_i \)
  
is a scalar field that measures the “spreading” of the vector field, \( g \), at \( x \). Divergence is also defined as an operation on a vector field, where it gives the tensor quantity \( \nabla \cdot g = \partial_i g_i \).

- **Curl:** \( \nabla \times g(x) = \left( \frac{\partial g_z}{\partial y} - \frac{\partial g_y}{\partial z} \right) \hat{x} + \left( \frac{\partial g_x}{\partial z} - \frac{\partial g_z}{\partial x} \right) \hat{y} + \left( \frac{\partial g_y}{\partial x} - \frac{\partial g_x}{\partial y} \right) \hat{z} \)
  
is a vector field that measures the “rotation” of a vector field, \( g \), at \( x \).

- **Laplacian:** \( \nabla^2 f(x) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \nabla \cdot \nabla f = \partial_i \partial_i f \)
  
is a second order derivative scalar field obtained from the scalar field \( f \) at \( x \).

- **Vector Laplacian:** \( \nabla^2 g(x) = \nabla^2 g_x \hat{x} + \nabla^2 g_y \hat{y} + \nabla^2 g_z \hat{z} \)
  
is a second order derivative vector field. A useful identity is that \( \nabla^2 g = \nabla(\nabla \cdot g) - \nabla \times \nabla \times g \).

**Signal Processing.** Because seismology is principally a study of signals that evolve with time, you will also be expected to understand and use basic analog and digital signal processing (e.g., filtering, spectra) concepts in this course. Much of the relevant content is covered in my Geophysics 505 course, which has notes available on the EES website.