1) A string segment of length $l$, fixed at both ends, is plucked so that its initial velocity is zero and its initial shape is the sinusoid

$$h \sin \left( \frac{2\pi x}{l} \right) \quad (0 \leq x \leq l) \quad (1)$$

where $h \ll l$.

The string end points at $x = 0$ and $x = l$ can be modeled as splices with an infinite linear density string segment.

a. What is the phase velocity of waves in this system if the tension is $\zeta$ and the linear density is $\rho$?

b. What will be the period, $T$ of the standing wave in (a)?

c. Plot the successive shapes of the resulting standing waves on the string over one period, $T$, in increments of $T/8$.

d. Produce a similar set of plots, except for an initial (triangle) shape of:

$$2hx/l \quad (0 \leq x \leq l/2) \quad (2)$$

$$2h(l - x)/l \quad (l/2 \leq x \leq l) \quad (3)$$

Hint: Use a Fourier series of standing waves from (a) to construct the indicated triangle wave from a suitable superposition of sinusoidal traveling wave components.

e. How many standing waves terms are necessary to achieve a wave shape accuracy at maximum amplitude that is everywhere less than 10% of the desired maximum amplitude, $h$? How about 1% of $h$?

f. Explain, and give mathematical expressions to show, why the modeling of this system becomes much more complicated when $h$ is not much smaller than $l$.

g. Fun extra credit – produce MATLAB movies showing the time evolution of your synthetic string-o-grams for (c) and (d).

You may email me your code if you wish.
2) For an ideal fluid, the constitutive relation between stress and strain is simply that of an isotropic medium, where $\mu \to 0$.

$$\sigma_{ij} = \kappa \Theta \delta_{ij}$$  \hspace{1cm} (4)

where $\kappa$ is the incompressibility (bulk modulus).

a. Derive the wave equation for propagating elastic waves in such a fluid by balancing the contact forces and the product of mass and acceleration in this medium.

b. Describe the particle motions for these elastic waves, showing the details of your derivation.

c. What is the approximate velocity of such waves in sea water? In air?