1) Calculate the frequency and phase response of the FIR filter corresponding to the `diff` operation in Matlab. Compare this response to that of exact differentiation.

2) Obtain a formula for the infinite number of FIR filter coefficients necessary to characterize a band-limited differentiator. This filter has the analog frequency response (for a unit sampling rate) of

\[
\Omega(f) = \begin{cases} 
2\pi f & (|f| \leq 1/\alpha) \\
0 & (|f| > 1/\alpha) 
\end{cases}
\]  

a) Write a Matlab program that calculates the FIR weights from your analytic formula.

b) Tabulate and plot the first 31 coefficients \((-15 \leq n \leq 15)\) for \(\alpha = 4\) and \(\alpha = 2\).

3) a) Write the bilinear \(z\) transform for an ideal broadband differentiator response, \(\Phi(s) = s\).

b) Obtain the corresponding difference equation in terms of \(y_n, y_{n-1}, x_n,\) and \(x_{n-1}\).

4) For \(\alpha = 2\), plot amplitude versus linear frequency/dB and linear phase versus linear frequency Nyquist interval \((-1/2 \leq f \leq 1/2)\) responses for:

   a) The bilinear \(z\) transform realization (from (3)),
   
   b) The 31 point FIR realization (from (2)), for the following cases:

   • Where the FIR filter weights are simply truncated (rectangular window).
   
   • Where the FIR filter weights are tapered using a Hamming window.

To facilitate easy comparisons above, use the same amplitude (dB; e.g., from -20 to 20 dB) and nonnegative frequency (linear; from 0 to 0.5) scales for all plots. Finally, overlay the ideal analog response. Label all curves appropriately.