# Effects of Boundary Condition on Shape of Flow Nets

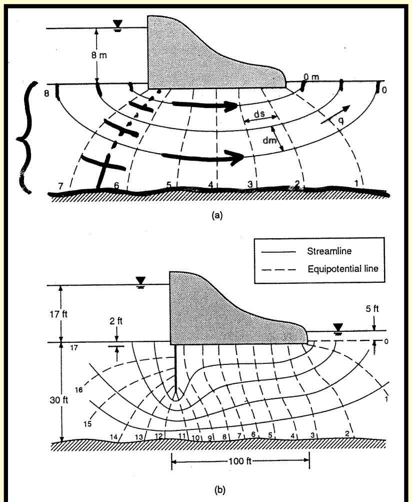
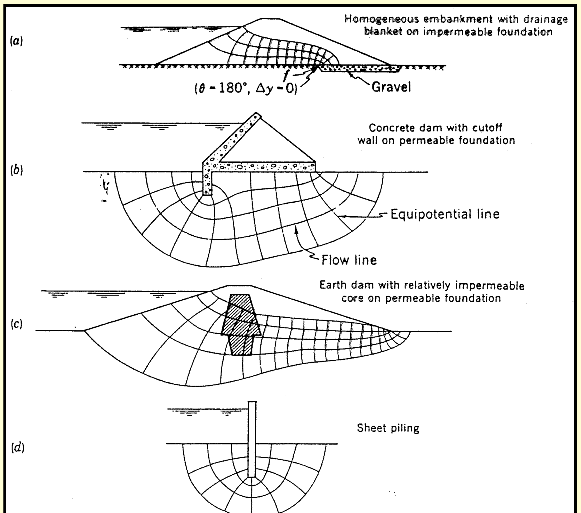



Figure 2.10 (a) Flow net for simple dam (section). (b) Flow net for complex dam (section). Source: Bedient and Huber, 1992.

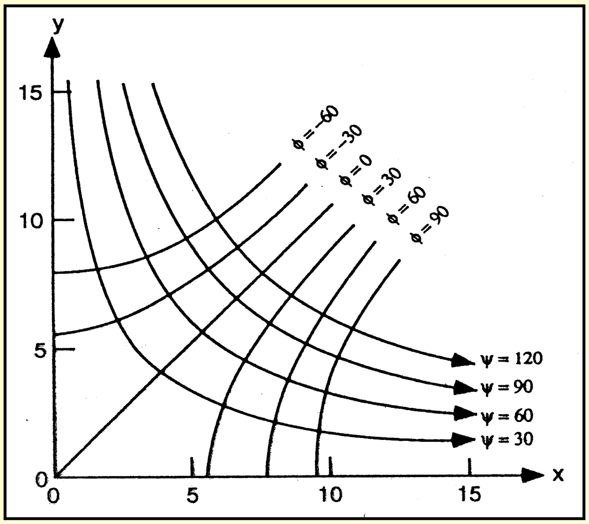
# Seepage and Dams




- Flow nets for seepage through earthen dams
- Seepage under concrete dams
- Uses boundary conditions (L & R)
- Requires curvilinear square grids for solution

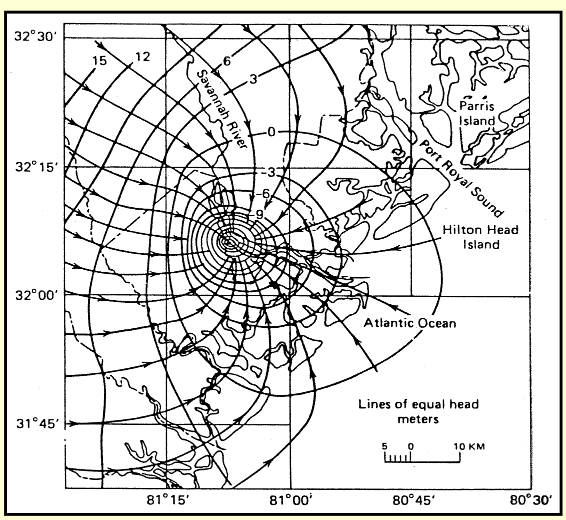
*After Philip Bedient Rice University*

# Flow Net in a Corner:



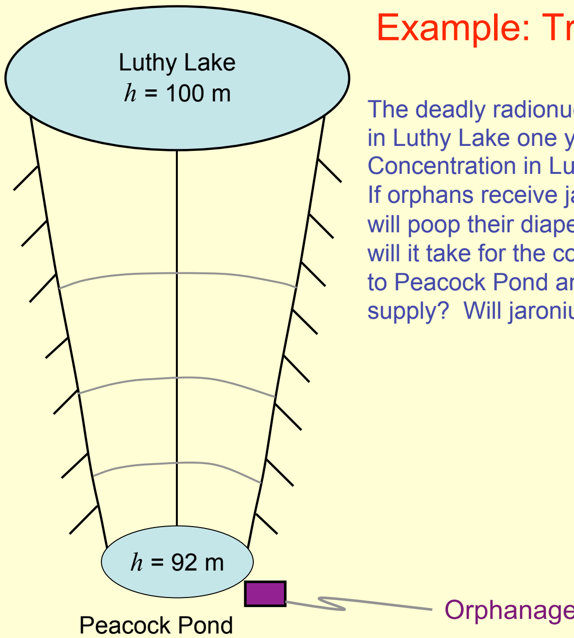

Streamlines Ψ are at right angles to equipotential Φ lines

# Flow to Pumping Center:




Contour map of the piezometric surface near Savannah, Georgia, 1957, showing closed contours resulting from heavy local groundwater pumping (after USGS Water-Supply Paper 1611).

## Example: Travel Time

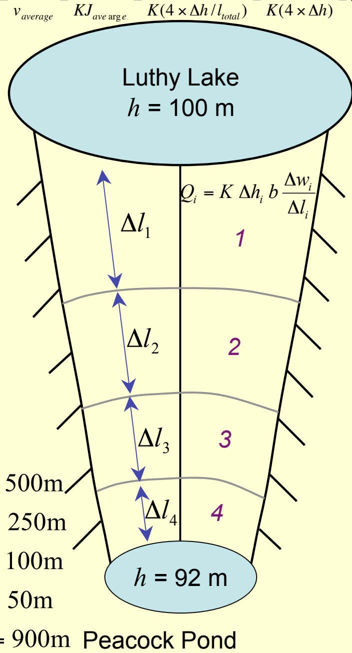



The deadly radionuclide jaronium was released in Luthy Lake one year ago. Half-life = 365 days
Concentration in Luthy Lake was 100 μg/L.
If orphans receive jaronium over 50 μg/L, they will poop their diapers constantly. How long will it take for the contaminated water to get to Peacock Pond and into the orphanage water supply? Will jaronium be over the limit?

$$\Delta t_{total\ simple} = \frac{l_{total}}{v_{average}} = \frac{nl_{total}}{KJ_{average}} = \frac{nl_{total}}{K(4\times\Delta h/l_{total})} = \frac{nl_{total}^2}{K(4\times\Delta h)} = 80 days << 131 days$$

## Travel time

### Contamination in Kidnapper Bog




$\Delta l_1 = 500\text{m}$
$\Delta l_2 = 250\text{m}$
$\Delta l_3 = 100\text{m}$
$\Delta l_4 = 50\text{m}$
$l_{total} = 900\text{m}$

$$\Delta t_{total} = \sum_1^4 \Delta t_i = \sum_1^4$$

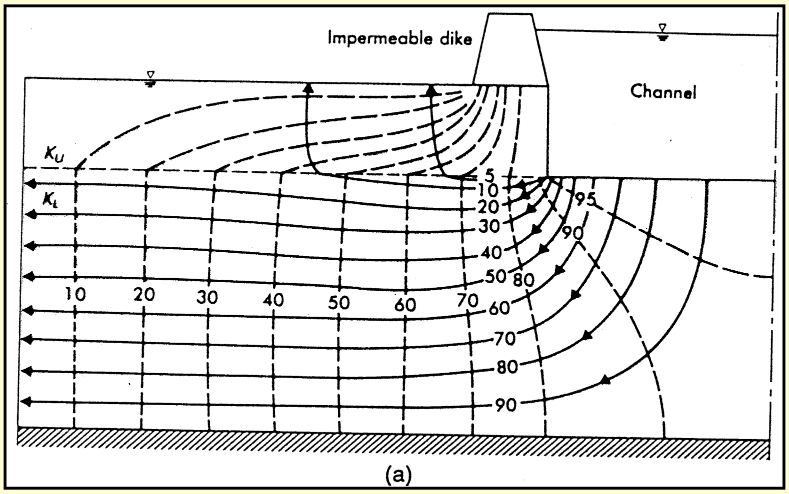
$$q_i = \frac{Q_i}{w_i b} = \frac{K\,\Delta h_i}{\Delta l_i} = \frac{K\,\Delta h}{\Delta l_i}$$

$$\Delta t_{total} = \sum_1^4 \Delta t_i =$$

Note: because of $\Delta l$ terms are squared, accuracy in estimating $\Delta l$ is very important.

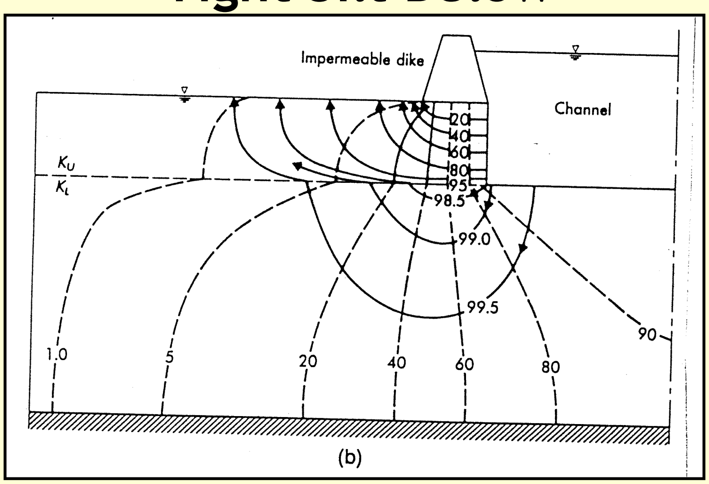
$n = 0.25$, $\Delta h = 2\text{m}$, $K = 10^{-5}\,\text{m/s}$,

# Two Layer Flow System with Sand Below




$K_u$ / $K_l$ = 1 / 50

# Two Layer Flow System with Tight Silt Below




Flow nets for seepage from one side of a channel through two different anisotropic two-layer systems. (a) $K_u$ / $K_l$ = 1/50. (b) $K_u$ / $K_l$ = 50. *Source: Todd & Bear, 1961.*

## Flow nets in anisotropic media

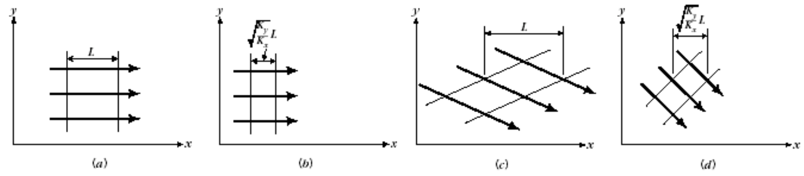
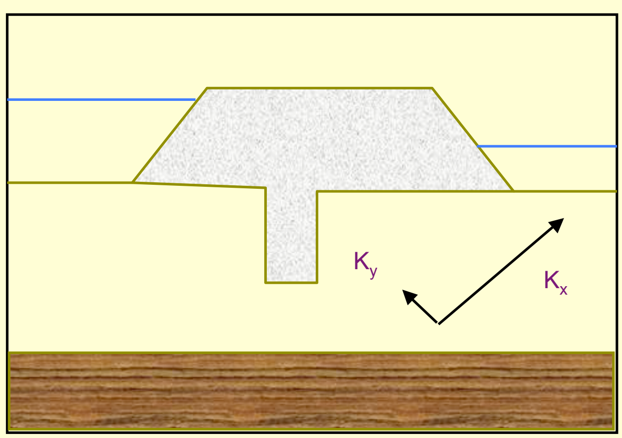



**Figure 5.11** Flownet in anisotropic media. In Panel (a), the direction of flow is parallel to the principal direction (x) of hydraulic conductivity: the horizontal dimension of the porous medium cross section is changed by a ratio of $\sqrt{K_y/K_x}$. In Panel (b), a transformed cross section with square nets is formed when the same ratio is applied to horizontal dimension. In Panel (c), the direction of flow is at some angle to the principal direction (x) of hydraulic conductivity: the angles formed by the streamlines and the equipotential lines are no longer 90 degrees. In Panel (d), a transformed cross section with square nets can be formed by applying the ratio of $\sqrt{K_y/K_x}$ to the horizontal dimension of the flownet. Note: arrows point to the direction of flow.

SZ2005 Fig. 5.11

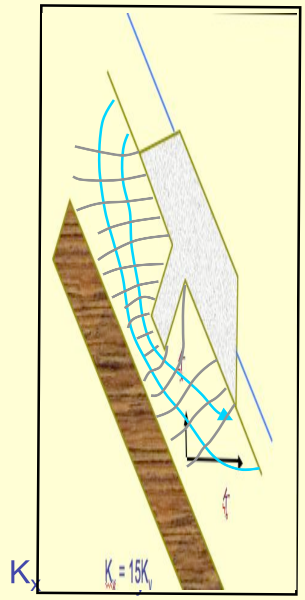
## Flownets in Anisotropic Media

Example:

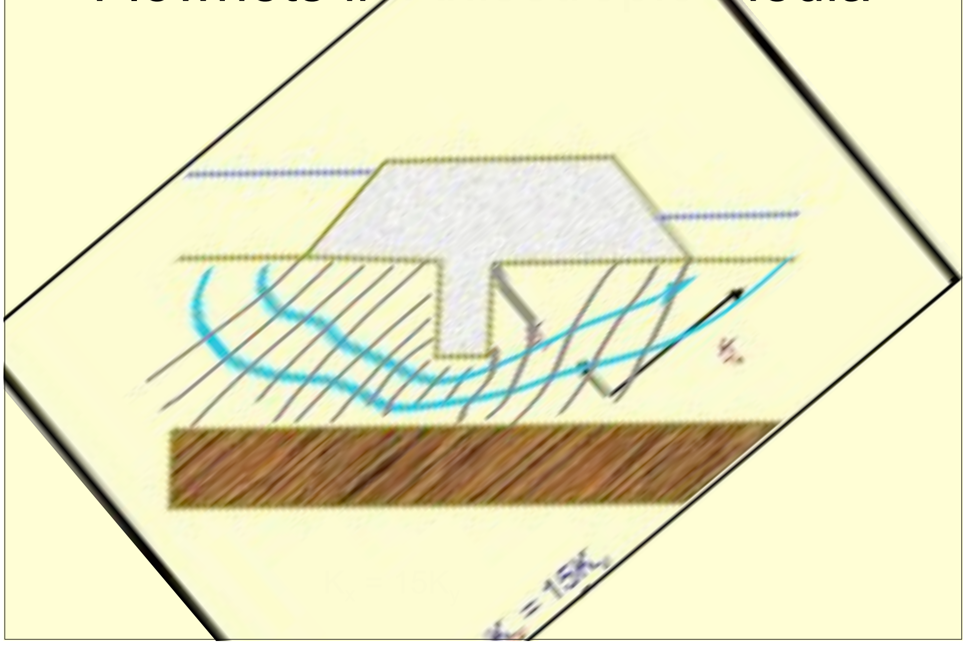



$K_x = 15K_y$

# Flownets in Anisotropic Media




# Flownets in Anisotropic Media



# Flow Nets: an example

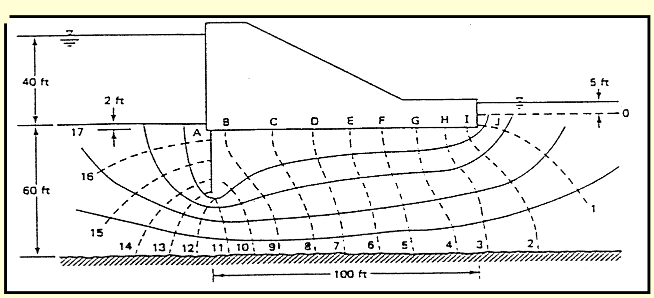
- A dam is constructed on a permeable stratum underlain by an impermeable rock. A row of sheet pile is installed at the upstream face. If the permeable soil has a hydraulic conductivity of 150 ft/day, determine the rate of flow or seepage under the dam.

*After Philip Bedient*
*Rice University*

# Flow Nets: an example

| Position: | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| Distance from front toe (ft) | 0 | 3 | 22 | 37.5 | 50 | 62.5 | 75 | 86 | 94 | 100 |
| n | 16.5 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1.2 |

The flow net is drawn with: m =          head drops =




*After Philip Bedient*
*Rice University*

# Flow Nets: the solution

- Solve for the flow per unit width:

$$q = m\,K \frac{\text{total change in head, } H}{\text{number of head drops}}$$

=

=

*After Philip Bedient*
*Rice University*

# Flow Nets: An Example

There is an earthen dam 13 meters across and 7.5 meters high.The Impounded water is 6.2 meters deep, while the tailwater is 2.2 meters deep. The dam is 72 meters long. If the hydraulic conductivity is $6.1 \times 10^{-4}$ centimeter per second, what is the seepage through the dam if the number of head drops is = 21

$K = 6.1 \times 10^{-4}$ cm/sec
= 0.527 m/day

*After Philip Bedient*
*Rice University*

# Flow Nets: the solution

From the flow net, the total head loss, *H*, is 6.2 -2.2 = 4.0 meters.

There are (*m*=) 6 flow channels and 21 head drops along each flow path:

Q = (*mKH/ number of head drops*) x dam length = (6 x 0.527 m/day x 4m / 21) x (dam length)

=

= for the entire 72-meter length of the dam

*After Philip Bedient*
*Rice University*

## Aquifer Pumping Tests

Why do we need to know T and S (or K and $S_s$)?

- To determine well placement and yield
- To predict future drawdowns
- To understand regional flow
- Numerical model input
- Contaminant transport

How can we find this information?

- Flow net or other Darcy's Law calculation
- Permeameter tests on core samples
- Tracer tests
- Inverse solutions of numerical models
- Aquifer pumping tests

# Steady Radial Flow Toward a Well

Aquifer Equation, based on assumptions becomes an ODE for $h(r)$ :

-steady flow in a homogeneous, isotropic aquifer $\nabla^2 h = 0$ (Laplace's equation)

⇩

-fully penetrating pumping well & horizontal, confined aquifer of uniform thickness, thus essentially horizontal groundwater flow $\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$ (Laplace's equation in $x,y$ 2D)

⇩

-flow symmetry: radially symmetric flow $\frac{1}{r}\frac{d}{dr}\left(r\frac{dh}{dr}\right) = 0$ (Laplace's equation in radial coordinates)

Boundary conditions

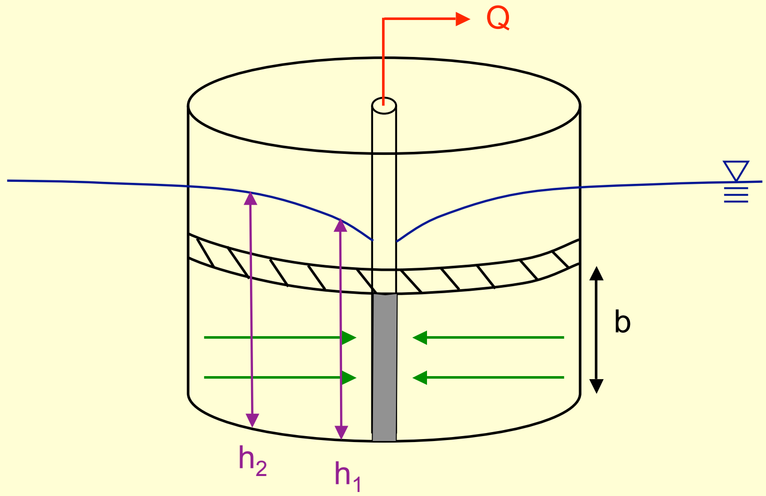
-2nd order ODE for $h(r)$ , need two BC's at two $r$'s, say $r_1$ and $r_2$

-Could be

- -one Dirichlet BC and one Neuman, say at well radius, $r_w$ , if we know the pumping rate, $Q_w$
- -or two Dirichlet BCs, e.g., two observation wells.

# Steady Radial Flow Toward a Well

Island with a confined aquifer in a lake




We need to have a confining layer for our 1-D (radial) flow equation to apply—if the aquifer were unconfined, the water table would slope down to the well, and we would have flow in two dimensions.

# Steady Radial Flow Toward a Well

ODE $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dh}{dr}\right) = 0$$

Multiply both sides by $r\ dr$

$$d\left(r\frac{dh}{dr}\right) = 0$$

Integrate

Evaluate constant $C_1$ using continuity.

The $Q_w$ pumped by the well must equal the total $Q$ along the circumference for every radius $r$.

$$Q_w = Q(r) = KA\frac{dh}{dr} =$$

$$= (2\pi r)Kb\frac{dh}{dr} =$$

Constant $C_1$: $Q_w = (2\pi r)T\frac{dh}{dr}$

$$r\frac{dh}{dr} = \qquad = C$$

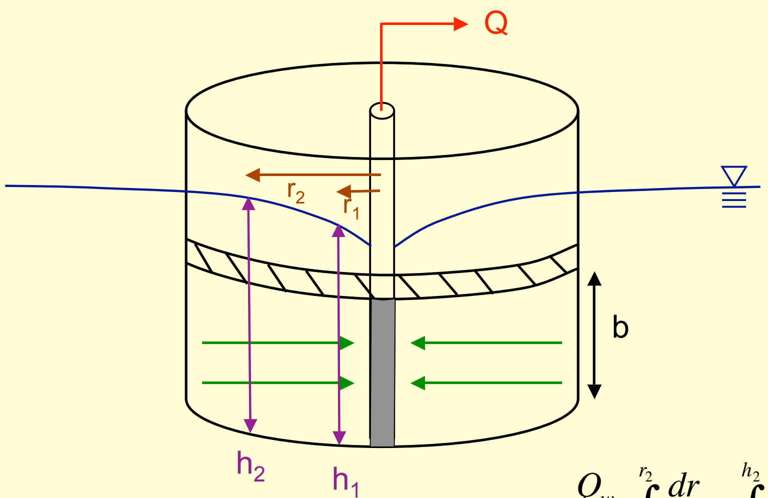
Separate variables:

$$\frac{Q_w}{2\pi T}\frac{dr}{r} = dh$$

Integrate again:

$$\frac{Q_w}{2\pi T}\int_{r_1}^{r_2}\frac{dr}{r} = \int_{h_1}^{h_2} dh$$

## Steady-State Pumping Tests




$$\frac{Q_w}{2\pi T}\int_{r_1}^{r_2}\frac{dr}{r} = \int_{h_1}^{h_2} dh$$

Integrate

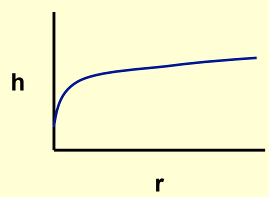
$$\frac{2\pi T}{Q}\left(h_2 - h_1\right) = \ln\left(\frac{r_2}{r_1}\right)$$

$T =$

Thiem Equation

## Steady-State Pumping Tests

Why is the equation logarithmic?
This came about during the switch to radial coordinates.




What is the physical rationale for the shape of this curve (steep at small r, flat at high r)?

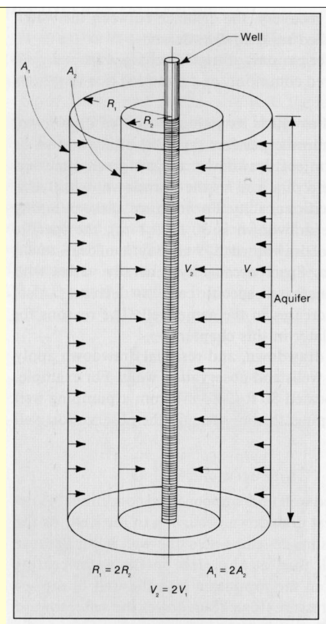



**Figure 9.2.** As flow converges toward a well, it passes through imaginary cylindrical surfaces that are successively smaller as the well is approached.

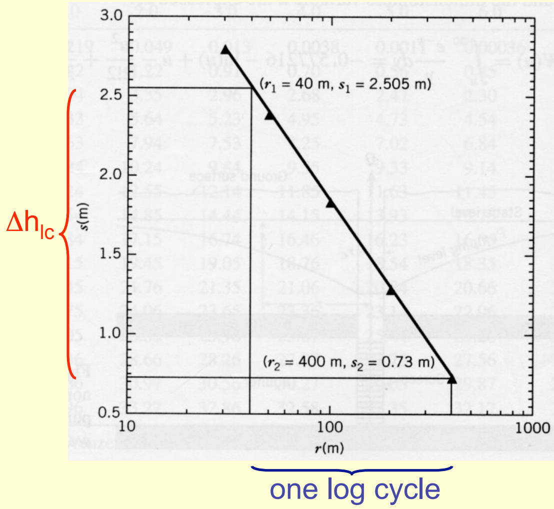
(Driscoll, 1986)

Think of water as flowing toward the well through a series of rings.

As we approach the well the rings get smaller. A is smaller but Q is the same, so $\frac{dh}{dl}$ must increase.

The Thiem equation tells us there is a logarithmic relationship between head and distance from the pumping well.

$$T = \frac{Q \ln\left(\frac{10 r_1}{r_1}\right)}{2\pi \, \Delta h_{lc}}$$



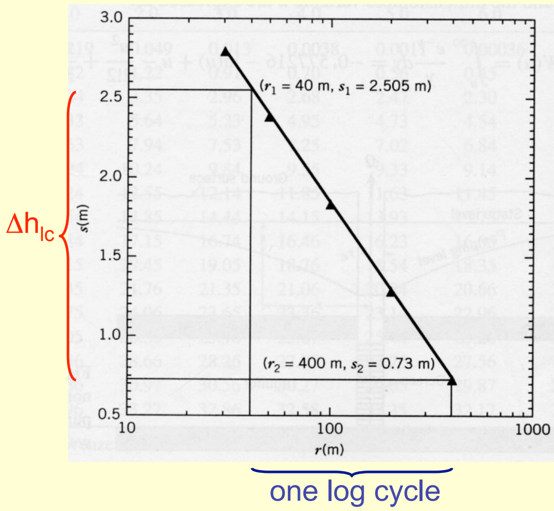

(Schwartz and Zhang, 2003)

The Thiem equation tells us there is a logarithmic relationship between head and distance from the pumping well.

$$T = \frac{Q \ln\left(\frac{10 r_1}{r_1}\right)}{2\pi \, \Delta h_{lc}}$$

$$\ln(x) \cong 2.3 \log(x)$$
$$\ln(10) \cong 2.303$$

$$T =$$




If T decreases, slope increases

(Schwartz and Zhang, 2003)

## Steady-State Pumping Tests

Can we determine S from a Thiem analysis?