Homework 4 - Spring 2008: Earth 441 – Aquifer Mechanics

Assigned: Feb 14, 2008 Due: Feb 21, 2008 at the beginning of class

Please make sure to show all your work, including any diagrams you used, in order to get full credit. This may be submitted in either paper or electronic format (Shasta@nmt.edu). Please cite your sources.

1. **Vector calculus and a flow in an aquifer** (15 pts)

Background:

The three-dimensional head in an aquifer can be calculated as a function of position such that:

$$h(x,y,z) = x^3 + 2y^2 - 3xz^2 - 2z^2 \qquad (1)$$

(a) Determine - ∇h, the three-dimensional vector of head gradient.
(b) Determine the divergence of the gradient in part *a*.
(c) Based on your answer to part *b*, what can we assume about a flow field that is adequately modeled by the *h(x,y,z)* in (1)? Is it isotropic or anisotropic? Steady state or unsteady flow? Homogeneous or heterogeneous? Is the specific-discharge field 1-,2-, or 3-dimensional? How can you tell any of these?

2. **Groundwater flow equations for saturated media** (24 pts)

Background:

The following assumptions are implicit in one or more of the forms of the groundwater flow equation listed below (Equations 1-7).

Assumptions:

Homogenous medium____

Isotropic medium____

Two-dimensional flow____

One-dimensional flow____

Steady-state conditions____

Constant fluid density____

$$S_s \frac{\partial h}{\partial t} = \left[\frac{\partial\left(K_x \frac{\partial h}{\partial x}\right)}{\partial x} + \frac{\partial\left(K_y \frac{\partial h}{\partial y}\right)}{\partial y} + \frac{\partial\left(K_z \frac{\partial h}{\partial z}\right)}{\partial z} \right] \tag{1}$$

$$S_s \frac{\partial h}{\partial t} = K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} \tag{2}$$

$$S_s \frac{\partial h}{\partial t} = K \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right] \tag{3}$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial x^2} = 0 \tag{4}$$

$$\frac{\partial h}{\partial t} = \frac{T}{S} \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right] \tag{5}$$

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{6}$$

$$\frac{\partial(n\rho)}{\partial t} = \nabla \cdot \left[\frac{\bar{k}\rho}{\mu} \left(\nabla p + \rho g \nabla z \right) \right] \tag{7}$$

Required:

(a) Next to each assumption, list the forms to which it applies, using the index number to the right of the equations
(b) Write Equations 1-4 in vector notation.
(c) What is the name of Equation 4?
(d) In addition to the assumptions implicit in Equation 7, state a scenario(s) under which it would be preferable to use the form of the groundwater flow given by Equation 7 as compared to the other equations.

Source: Modified from Ingebritsen et al., 2006, Problem 1.4

3. **Homogeneous and isotropic flow net with calculations** (17 pts)

Required: Draw a flow net for seepage through the earthen dam shown in Figure 2. Assume homogeneous and isotropic conditions. If the hydraulic conductivity of the material used in the dam is 0.22 ft/day, what is the seepage per unit width per day?

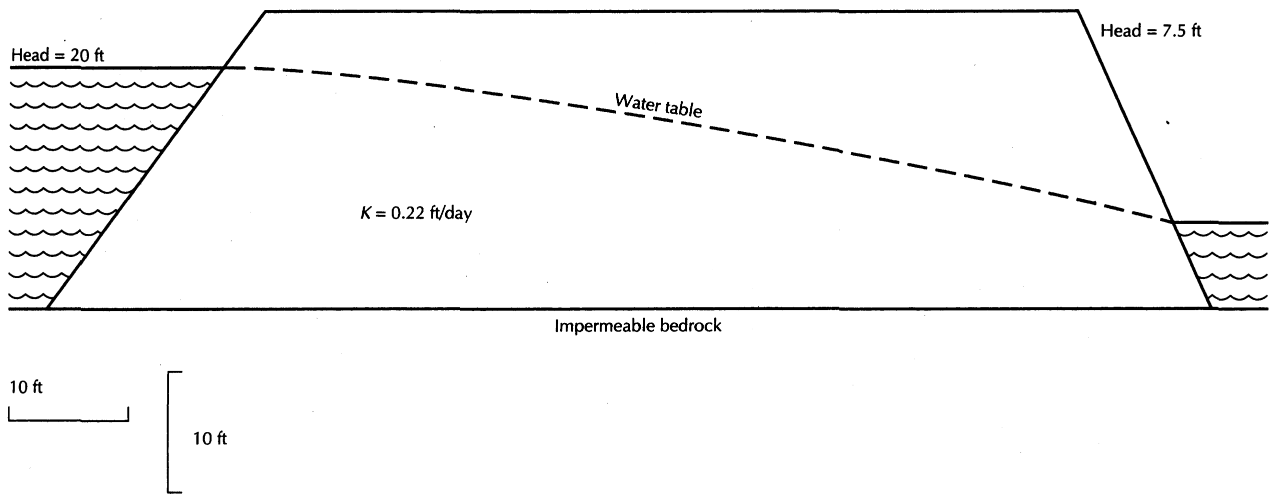

Figure 2. Diagram of an earthen dam for the construction of a flow net.

Source: Fetter, 2001, Problems 17

4. **Flow net for homogeneous and isotropic conditions** (15 pts)

Background: Figure 1 shows a dike beside a flow channel. The line *IJ* is the bottom of the channel. The dike *ABCEF* and the sheet *BC* are very-low-permeable material. The line *EK* is the water level in the aquifer, and the line *GH* is the water level in the channel.

Required: Draw a flow net that depicts the pattern of flow under the dike. Assume homogenous and isotropic conditions for the aquifer.

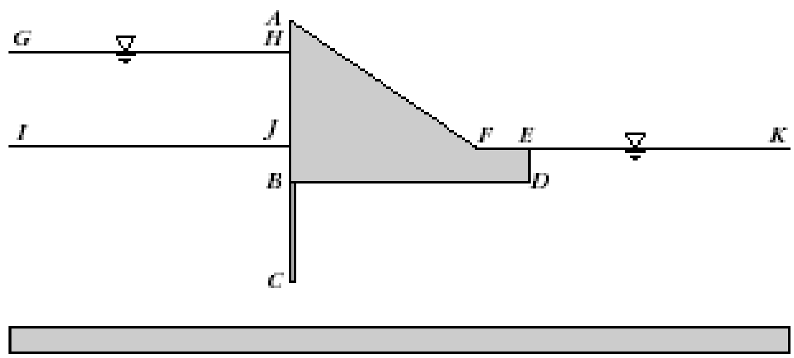

Figure 1. Dike along a flow channel.

Source: Schwartz, 2003, Exercise 5.2

5. **Flow nets for homogeneous, isotropic, and anisotropic conditions** (18 pts)

Background:

Figure 3 is a map view of an irregularly shaped aquifer bounded by two streams and two no-flow boundaries. The principle directions of hydraulic conductivity are *x* and *y*.

Required:

(a) Assume that $K_y/K_x = 1$ and transmissivity of the aquifer is 3,000 m^2/day; draw a flow net for the aquifer and calculate the total rate of flow from stream AB to stream CD.
(b) Assume that $K_y/K_x = 1/9$, and draw a flow net for the aquifer.
(c) Assume that $K_y/K_x = 4$, and draw a flow net for the aquifer.

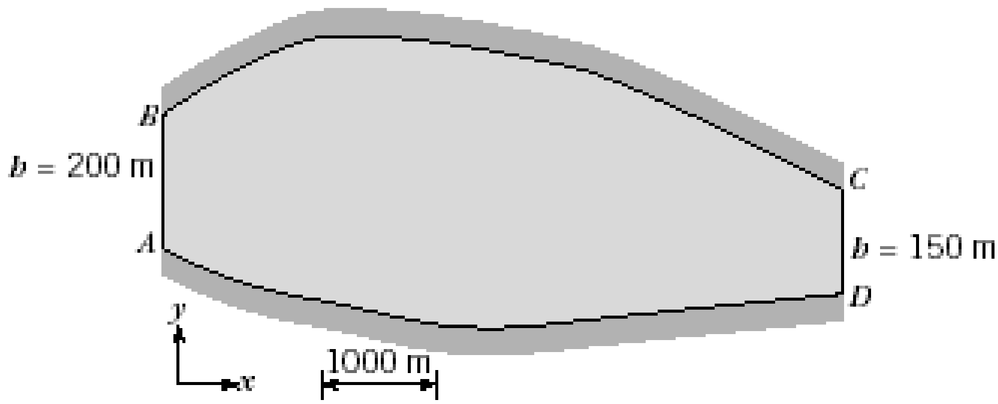

Figure 3. Map view of a buried valley aquifer.

Source: Schwartz and Zhang, 2003, Exercise 5.3

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6. **Flownet for groundwater flow in an unconfined aquifer** (16 pts)

Background:

In a water-table aquifer bounded on the ends by rivers, the hydraulic conductivity is 50 m/day. The hydraulic heads at x = 0 and x = 1000 m are 100 m and 90 m, respectively.

Required:

(a) Draw your own diagram and use flow nets to calculate the flow rate in a unit width of the aquifer.
