

Theoretical Seismology (GEOP 523) Homework 3; Due 3/21/10

February 16, 2011

1) Consider an inhomogeneous isotropic medium where the Lamé elastic parameters are functions of position. In this case the spatial derivatives of stress found from differentiating the constitutive relationship will now include terms that are proportional to spatial derivatives of the elastic moduli.

a) Show, that the appropriate expression for the equation of motion for inhomogeneous isotropic media, i.e., where

$$\sigma_{ij} = \lambda(\mathbf{x})\Theta\delta_{ij} + 2\mu(\mathbf{x})\epsilon_{ij}$$

and $\rho = \rho(\mathbf{x})$ is

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} = \mu \nabla^2 u_i + (\lambda + \mu) \frac{\partial \Theta}{\partial x_i} + \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial \mu}{\partial x_j} + \Theta \frac{\partial \lambda}{\partial x_i}. \quad (1)$$

Consider the case where the elastic moduli and the density vary exponentially in the \hat{x}_3 direction as

$$\lambda(\mathbf{x}) = \lambda_0 e^{\alpha x_3} \quad (2)$$

$$\mu(\mathbf{x}) = \mu_0 e^{\alpha x_3} \quad (3)$$

$$\rho(\mathbf{x}) = \rho_0 e^{\alpha x_3}, \quad (4)$$

where α is a positive constant.

For plane body waves propagating in the \hat{x}_3 direction with displacements in the \hat{x}_2 and \hat{x}_3 directions:

b) Verify that your solutions approach the homogeneous medium solutions when the inhomogeneity parameter α approaches zero.

c) Discuss if such plane waves maintain their amplitude with increasing x_3 .

d) Discuss effects due to changing the frequency/wavelength of the waves with respect to the scale length α^{-1} .

e) Determine and discuss if waves of arbitrarily long wavelength can propagate in this medium.

Hint: This medium will be dispersive (velocity will be a function of ω). Substitute general P- and S-wave harmonic plane wave displacement solutions propagating in the \hat{x}_3 direction,

$$u_3 = C e^{i(\omega t - k x_3)} \hat{x}_3 \quad (5)$$

and

$$u_2 = C e^{i(\omega t - k x_3)} \hat{x}_2 \quad (6)$$

respectively, and solve for a complex wavenumber, k , which will be a function of ω , α , and the $x_3 = 0$ body wave velocities

$$v_{s0} = \sqrt{\frac{\mu_0}{\rho_0}} \quad (7)$$

$$v_{p0} = \sqrt{\frac{\lambda_0 + 2\mu_0}{\rho_0}}. \quad (8)$$

2) We customarily treat the Earth's surface as a free surface. However, the atmosphere or course propagates P (sound) waves.

a) Calculate the P to P transmission coefficient into an atmosphere at standard temperature and pressure as a function of Earth density and P wave velocity.

What are reasonable transmission coefficients for vertically-traveling:

b) 60 Hz P waves in Socorro near NMT?

c) 5 s period P waves in a continent?

d) 1 s period P waves in the oceans?

e) What is the pressure variation corresponding to 60 Hz atmospheric P waves at the standard threshold of hearing (0 dB)? What must the corresponding 60 Hz P-wave displacement amplitudes be for vertically-traveling P waves in a soil with a velocity of 300 m/s and a density of 1.3 to produce this level of atmospheric excitation?