Data Processing and Analysis (GEOP 505)

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Plotting Spectra Using Decibels

Because the amplitudes describing the spectral responses of physical systems in nature as well as filters and instruments, frequently span many orders of magnitude, amplitude responses are frequently plotted as a function of frequency using either as log-linear or log-log displays. The standard way to do this is using a *decibel* (dB) scale. The fundamental need for a logarithmic scale arises for the same reason as the earthquake of astronomical magnitudes, or for sound intensity (a 'Bel' was originally a unit of sound intensity, after Alexander Graham Bell); the need to display many orders of magnitude on a single plot.

The decibel relationship between two values is defined as

$$d = 20 \log_{10} \left| \frac{\alpha_1}{\alpha_2} \right| \tag{1}$$

where α_2 is a reference level. In examining system responses, α_2 in (1) is commonly taken to be unity, so that 0 dB corresponds to unit gain. An amplitude change of a factor of four is equal to approximately 12 dB, a factor of two is equal to about 6 dB, a factor of $\sqrt{2}$ is equal to about 3 dB, and so forth.

Decibels are also conveniently used to express rates of exponential falloff in a system response. This is especially common in engineering specifications. For example, a response that is proportional to $1/f$ (e.g., a single-pole system with no zeros such as a simple RC low-pass analog filter) decays at (approximately) $6 \approx 20 \log_{10}(2)$ dB per octave (per frequency doubling), or at $20 = 20 \log_{10}(10)$ dB per decade (per 10-fold frequency increase)

vs. log frequency plot, such asymptotic power law behavior is easy to predict (and sketch), because a falloff of f^{-n} is just a straight line with a slope of $20n$ dB/decade. One can thus approximately sketch the amplitude response of system as a set of simple lines with differing slopes (such plots are called *Bode plots*).

Confusion can sometimes arise when considering power or intensity ratios. The convention is to convert to amplitudes before calculating decibels, i.e.,

$$d = 20 \log_{10} \left(\frac{e_1}{e_2} \right)^{\frac{1}{2}} = 10 \log_{10} \left(\frac{e_1}{e_2} \right) \ . \tag{2}$$

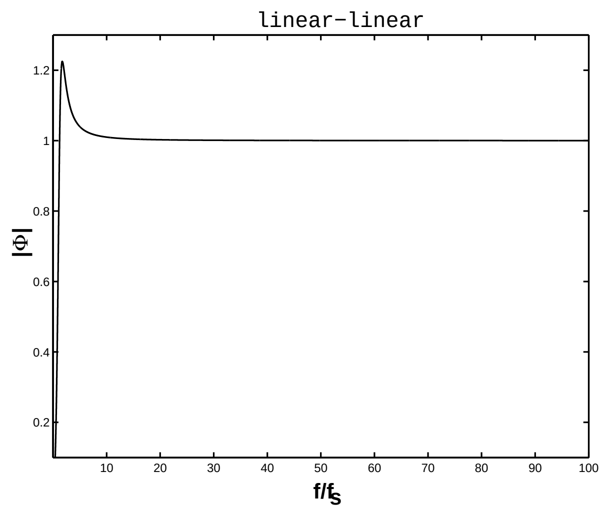

Figure 1: Linear-linear plot of the amplitude response of a seismometer, $\zeta = \omega_s/\sqrt{2}$.

Power spectra, for example, are thus plotted using

$$d = 10\log_{10}\left|\frac{\alpha_1}{\alpha_2}\right| \tag{3}$$

rather than $20\log_{10}$ (1), and the common PSD units are dB relative to 1 U^2/Hz. Similarly, the sound decibel scale uses (3), as it is based on intensity (the reference level is a power, $\alpha_2 = 10^{-12}$ W/m^2, judged to be the approximate threshold of hearing).

Figures 1, 2, and 3 show the amplitude displacement-displacement response of an underdamped seismometer with $\zeta = \omega_s/\sqrt{2}$ in linear-linear, dB (log)-linear, and db-log plots, respectively. Note that the 3 plot is most easily interpretable, shows the essential characteristics of $|\Phi|$ most clearly, and has the expected quadratic response fall-off of 40 dB/decade at low frequencies.

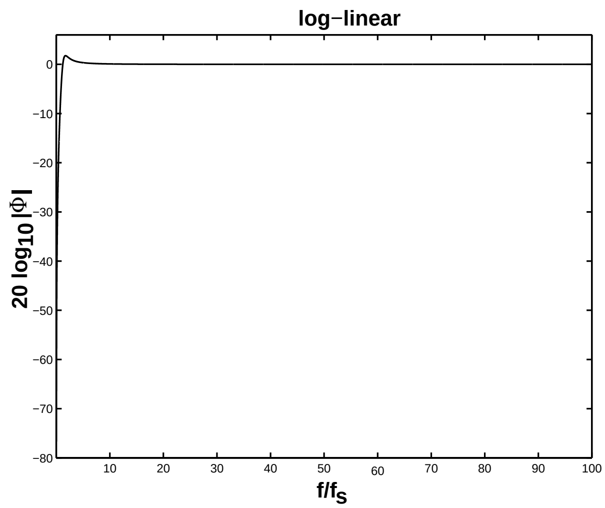

Figure 2: Log-linear plot of the amplitude response of a seismometer, $\zeta = \omega_s/\sqrt{2}$.

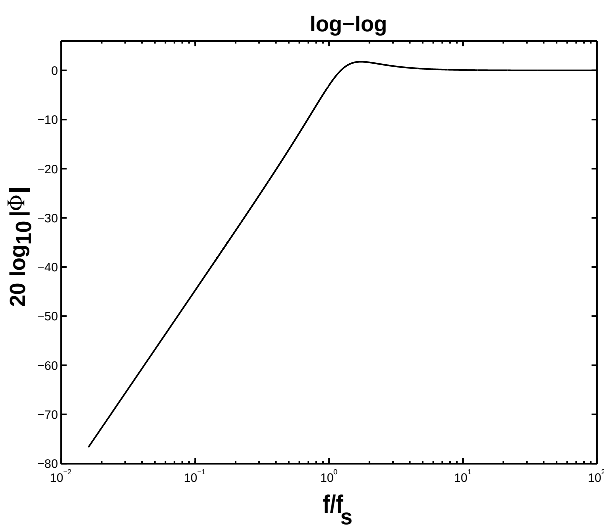

Figure 3: Log-log plot of the amplitude response of a seismometer, $\zeta = \omega_s/\sqrt{2}$.