Spring, 2005 Data Processing and Analysis

Homework 5; Due 5/2/05

April 18, 2005

1) The topographic relief of a region is found to be well-approximated by a random process where the ground elevation in profile is found to be a sample from a normal process

$$y(x) \;=\; Y(x) \;+\; y_0 \tag{1}$$

where Y is a real-valued random stationary process, x is the horizontal distance from the origin and y_0 is the mean elevation above sea level.

The PSD of Y is found to be

$$h_Y(k) \;=\; \pi a^2 b e^{-2\pi b|k|} \tag{2}$$

where a and b are real positive constants and k is the spatial frequency.

a) Find the autocovariance function for Y. Give the units of a and b and provide a physical interpretation.

b) Using your algorithm from (1), generate and plot two realizations of this 2-d random landscape for $b/a \;=\; 10$. Be sure to include a long enough series so that long wavelength behavior is visible and to use a high enough spatial sampling rate to avoid aliasing.

c) If $y_0 \;=\; 3.5a$, what fraction of the profile is expected to be below sea level?

d) Describe how your procedure could be generalized using 2-d filtering to construct realizations of a 3-dimensional pseudo-landscape.

e) Generate two 3-d landscape realizations using your procedure for $b/a \;=\; 10$. Use Matlab's contour or surf utilities to display your result.

2) Suppose that a 400 km x 400 km swath of a distant planetary landscape is given by the file landscape.mat (available from the class web site), which has a sampling spacing of 1 km. This landscape is recorded by a remote sensing instrument that has a resolution scale length of around 15 km, so that its impulse response is given by:

$$\phi(r) = e^{-r^2/15000^2} \tag{3}$$

where

$$r = \sqrt{x^2 + y^2} \tag{4}$$

is the distance in meters. Apply this filter to the landscape using a 2-dimensional FIR implementation and graphically compare the two surfaces.