Spring, 2005 Data Processing and Analysis (GEOP 505/Math 587)

Homework 4; Due 4/1/05

March 18, 2005

1) Obtain a formula for the infinite number of FIR filter coefficients necessary to characterize a band-limited differentiator, which has the analog frequency response for a unit sampling rate of

$$\Omega(f) \;=\; \begin{cases} 2\pi i f & (|f| \le 1/\alpha) \\ 0 & (|f| > 1/\alpha) \end{cases} \tag{1}$$

2) a) Write a MATLAB program that calculates the FIR weights.

b) Tabulate and plot the first 31 coefficients ($-15 \le n \le 15$) for $\alpha = 4$ and $\alpha = 2$.

3) a) Write the bilinear z transform for an ideal broadband differentiator response, $\Phi(s) = s$.

b) Obtain the corresponding difference for (3a) equation in terms of y_n, y_{n-1}, x_n, and x_{n-1}.

4) For $\alpha = 2$, plot dB-log f Nyquist interval amplitude *and* phase responses as functions of frequency for:

a) The bilinear z transform realization (from (3)),

b) The 31 point FIR realization (from (2)); do this for two cases

i) Where the FIR filter weights are simply truncated (rectangular window)

ii) Where the FIR filter weights are tapered with a Hamming window.

To facilitate easy comparison, use the same amplitude (dB scale; from -20 to 20 dB) and frequency scales for all plots. Overplot the analog response from (1) to compare the digital and analog response curves.

5) Compare your differentiator responses to the Matlab *diff* operation response in the frequency domain. Quantitatively characterize how good *diff* is at approximating the differentation operation is in phase and amplitude.