Spring, 2005 Data Processing and Analysis

Homework 3; Due 03/07

February 23, 2005

1) a) Show that the N-point DFT of a real sequence, x_n is Hermitian, i.e.,

$$X_{N-k} = X_k^* \ . \tag{1}$$

b) Show that the N independent input numbers produce only $N/2+1$ independent output numbers (N even) or $(N-1)/2+1$ independent output numbers (N odd) in this case. Why hasn't information been lost?

2) Estimate Power Spectral density for the 100 sample/s velocity time series *mysteryseries.ascii* posted on the class site. The units of the time series are m/s.

a) Use a data length of $N_1 = 2^{14} = 16384$ samples (163.84 s).

b) Use a data length of $N_2 = 2^{16} = 65536$ samples (655.36 s).

c) Use a data length of $N_3 = 2^{18} = 262144$ samples (2621.44 s).

In each case, estimate the PSD using the *pwelch* MATLAB function with 16 subwindows (NFFT = $N/16$). Plot your one-sided (nonnegative frequency) PSD estimates on a decibels vs. log (base 10) frequency scale. Title your plots appropriately, and show the proper units for the PSD and frequency axes.

d) For (c), calculate total signal power in the time and frequency (PSD) domains by doing appropriate sums, and show that they are equivalent.

e) Estimate the relative signal amplitudes of any narrow-band spectral components that you observe in (c).

f) Using the convolution theorem, convolve the mystery time series with the displacement-to-displacement response of an underdamped $\omega_s = 4\pi$ Hz ($f_s = 2$ Hz) seismometer with damping $\zeta = \omega_s/\sqrt{2}$. Explain the observed changes in the PSD using the full N_3 samples of data, relative to the result of (c).

Extra Credit:

Evaluate (c) above using the MATLAB *pmtm* multitaper PSD estimation program, the full signal length of $N_3 = 2^{18}$ samples, and a time-bandwidth product of 4. Compare with results obtained using Welch's method and explain any differences/improvement/interpretations. You may need to find a fast computer with lots of memory!