Spring, 2005 Analysis of Time Series and Spatial Data

Homework 2; Due 2/18

February 9, 2005

(1) If the frequency response of a linear system is
$\Phi(f) = \alpha(f) + i\beta(f)$
and is Hermitian, i.e.,
$\Phi(f) = \Phi^*(-f)$
show that
$\phi(t) * \sin(2\pi ft) = |\Phi(f)| \cdot \sin(2\pi ft + \theta(f))$
where
$\theta(f) = \tan^{-1}\left(\frac{\beta(f)}{\alpha(f)}\right)$
and
$|\Phi(f)| = (\alpha^2(f) + \beta^2(f))^{\frac{1}{2}}$

(2) In studying earthquake sources, it is especially desirable to have a highly accurate record of the true displacement history of the ground

a) Write a MATLAB program that simulates and plots the first five seconds of the time-domain displacement response of three vertical seismometers with damping coefficients of $\zeta = 0.707\omega_s$ and natural free resonant periods of $T_s = 0.1$ s, 1.0 s, and 100 s, respectively, when the true displacement of the ground is that given in the 100 sample/s file *xtruedisp.ascii* (available on the web site). Use a sampling rate of 100 samples/s througout your time-domain calculations (the conv and diff functions in Matlab will come in handy here).

b) Comparing the true and detected ground displacements, what do you conclude about the ability of the three seismometers to say meaningful things about very-long-period or permanent ground offsets following large earthquakes; can you suggest a frequency threshold below which we are unlikely to uncover useful information in each case.

c) Use your program to plot the time domain response of the three seismometers to a displacement step input; explain the results.

(3) a) Use Matlab to estimate and plot the equilibrium crustal topography response beneath and near a North American Pleistocene ice sheet lobe with a width of 200 km, a semicircular profile, a maximum elevation of 2 km, and an approximate density (assumed uniform throughout) of $\rho_l = 1000$ kg/m^3. Assume that the rigid lithosphere is 100 km thick and is uniform with Poisson's ratio $\nu = 0.25$, and Young's modulus $E = 5 \times 10^{10}$ Pa. Also assume that the lithosphere floats on a displaced uniform mantle with density $\rho_m = 3.3$.

b) What is the spatial wavelength for a load in this region for which the buoyant compensation is 50%?